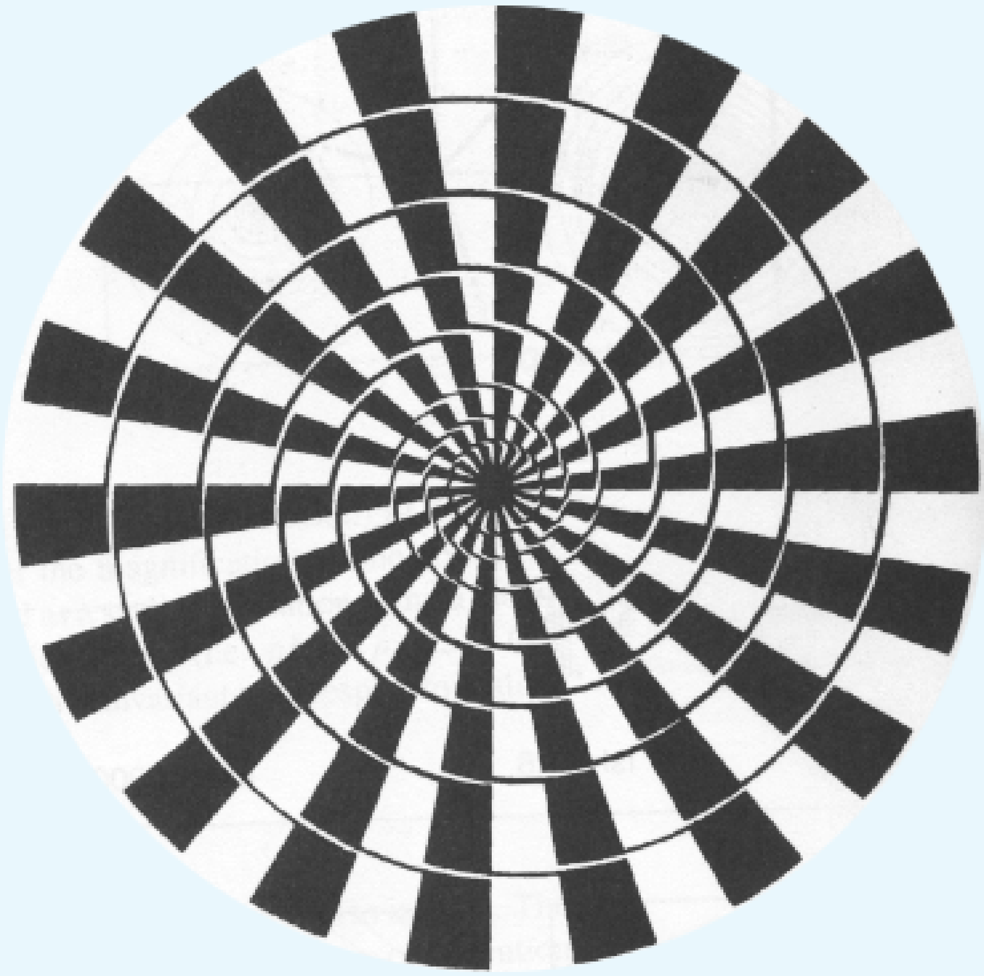

Julia Fractals in PostScript

Kees van der Laan

In Memory of Hans Lauwerier

Contents

Quite something

If only you'll remember . . .

If only you'll remember

- convergence
  versus
  bifurcation, chaos

If only you'll remember

- convergence
  versus
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- Mandelbrot fractal
  is map for
  Julia fractals

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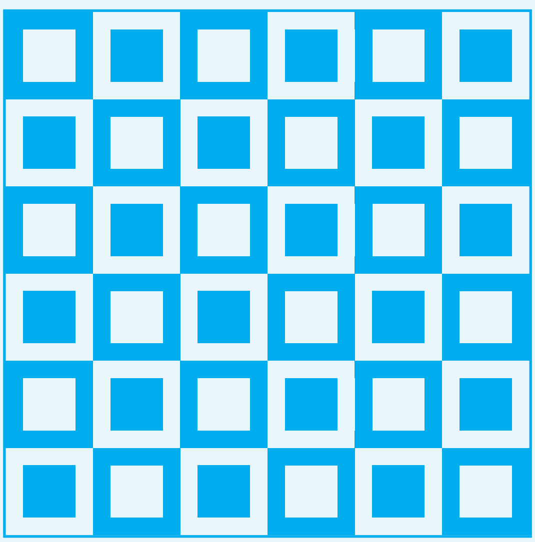
I'll  be happy

XaoS movie I II

Motivation

Motivation
- How to draw Fractals?

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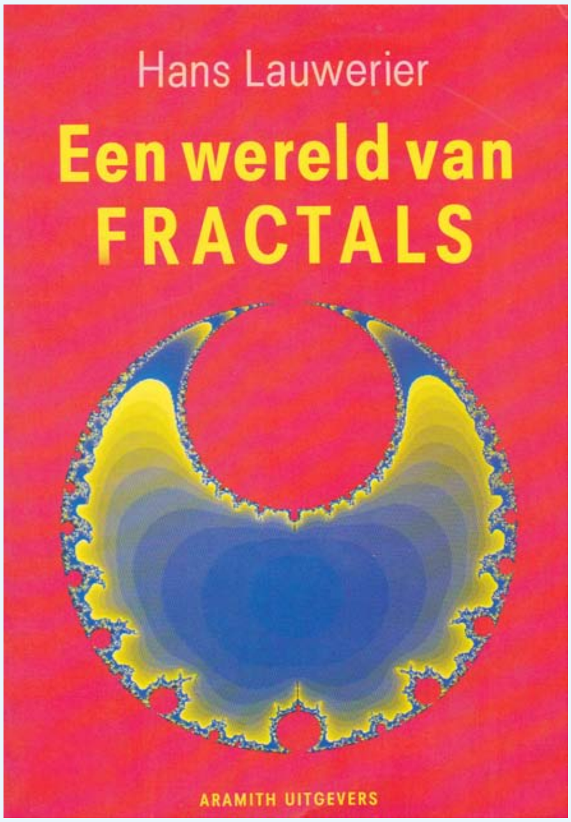
- How to draw Fractals?
- Lauwerier's BASIC codes



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- How to draw Fractals?
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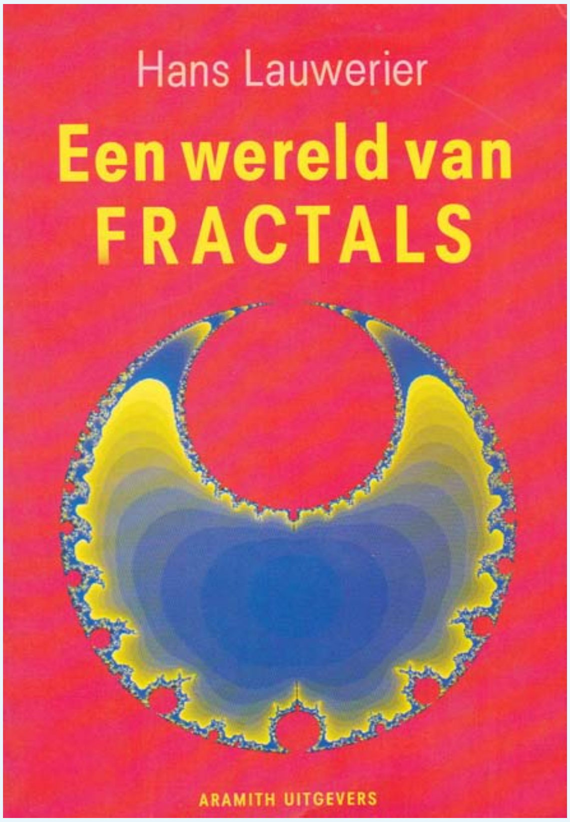

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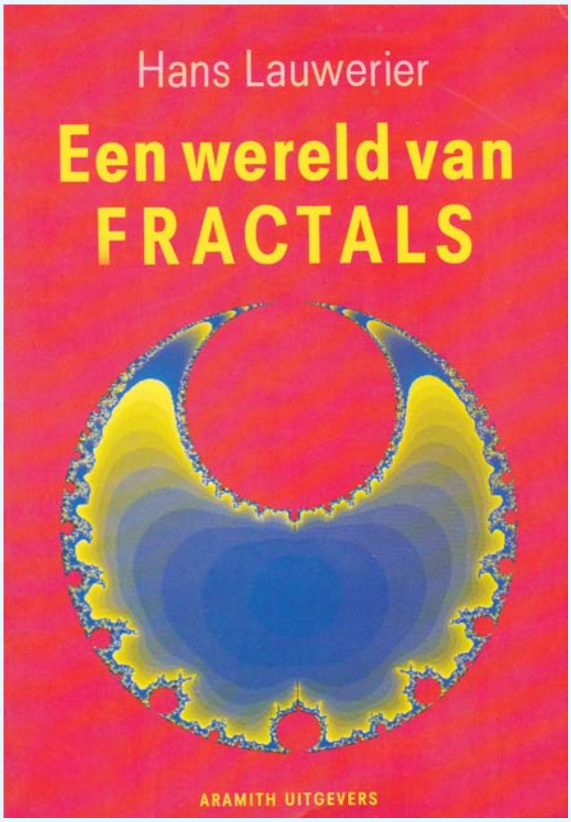

- Xaos, Fractalus ... packages

Gaston Julia 1893-1978

Gaston Julia 1893-1978

Mémoire sur l'itération des fonctions rationnelles 1918

Gaston Julia 1893-1978

Mémoire sur l'itération des fonctions rationnelles 1918

Fractals avant la lettre

Gaston Julia 1893-1978

Mémoire sur l'itération des fonctions rationnelles 1918

Fractals avant la lettre

Made famous by

Mandelbrot 1980

Julia dynamical system

Julia dynamical system

$$z_{i+1} = f(z) = z_i^2 + c,\ z_i, c \in \mathbb{C}\ i = 0, 1, 2, \ldots$$

Julia dynamical system

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Julia dynamical system

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Stability $|f'(l_{1,2})| < 1$

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Attractor Infinity

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Strange attractors

Julia dynamical system

$$z_{i+1} = f(z) = z_i^2 + c,\ z_i, c \in \mathbb{C}\ i = 0, 1, 2, \ldots$$

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Stability $|f'(l_{1,2})| < 1$

Attractor Infinity

Strange attractors

Julia fractal is repeller

For the unwary

For the unwary

$$z_{i+1} = f(z) = z_i^2 - .5 \qquad i = 0, 1, 2, \ldots$$

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Fixed-points

$l_1 = .5(1 + \sqrt{3}) \approx 1.366$

$l_2 = .5(1 - \sqrt{3}) \approx -.366$

For the unwary

$$z_{i+1} = f(z) = z_i^2 - .5 \qquad i = 0, 1, 2, \ldots$$

Fixed-points

$l_1 = .5(1 + \sqrt{3}) \approx 1.366$

$l_2 = .5(1 - \sqrt{3}) \approx -.366$

Stability

$|f'(l_1)| = |1 + \sqrt{3}| \approx 2.732$ **repeller**

$|f'(l_2)| = |1 - \sqrt{3}| \approx 0.732$ **attractor**

For the unwary

$$z_{i+1} = f(z) = z_i^2 - .5 \qquad i = 0, 1, 2, \ldots$$

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For the unwary

But . . . what if c=-1

For the unwary

$$z_{i+1} = f(z) = z_i^2 - 1 \qquad i = 0, 1, 2, \ldots$$

For the unwary

$$z_{i+1} = f(z) = z_i^2 - 1 \qquad i = 0, 1, 2, \ldots$$

Fixed-points

$$l_1 = .5(1 + \sqrt{5}) \approx 1.618 \qquad \phi$$

$$l_2 = .5(1 - \sqrt{5}) \approx -.618 \qquad -1/\phi$$

For the unwary

$$z_{i+1} = f(z) = z_i^2 - 1 \qquad i = 0, 1, 2, \ldots$$

Fixed-points

$l_1 = .5(1 + \sqrt{5}) \approx 1.618 \qquad \phi$

$l_2 = .5(1 - \sqrt{5}) \approx -.618 \qquad -1/\phi$

Stability

$|f'(l_1)| = |1 + \sqrt{5}| \approx 3.236$ **repeller**

$|f'(l_2)| = |1 - \sqrt{5}| \approx 1.236$ **repeller**

For the unwary

$$z_{i+1} = f(z) = z_i^2 - 1 \qquad i = 0, 1, 2, \ldots$$

Fixed-points

$$l_1 = .5(1+\sqrt{5}) \approx 1.618 \qquad \phi$$

$$l_2 = .5(1-\sqrt{5}) \approx -.618 \qquad -1/\phi$$

Stability

$|f'(l_1)| = |1+\sqrt{5}| > 1$ **repeller**

$|f'(l_2)| = |1-\sqrt{5}| > 1$ **repeller**

For the unwary

$$z_{i+1} = f(z) = z_i^2 - 1 \qquad i = 0, 1, 2, \ldots$$

Fixed-points

$l_1 = .5(1 + \sqrt{5}) \approx 1.618 \qquad \phi$

$l_2 = .5(1 - \sqrt{5}) \approx -.618 \qquad -1/\phi$

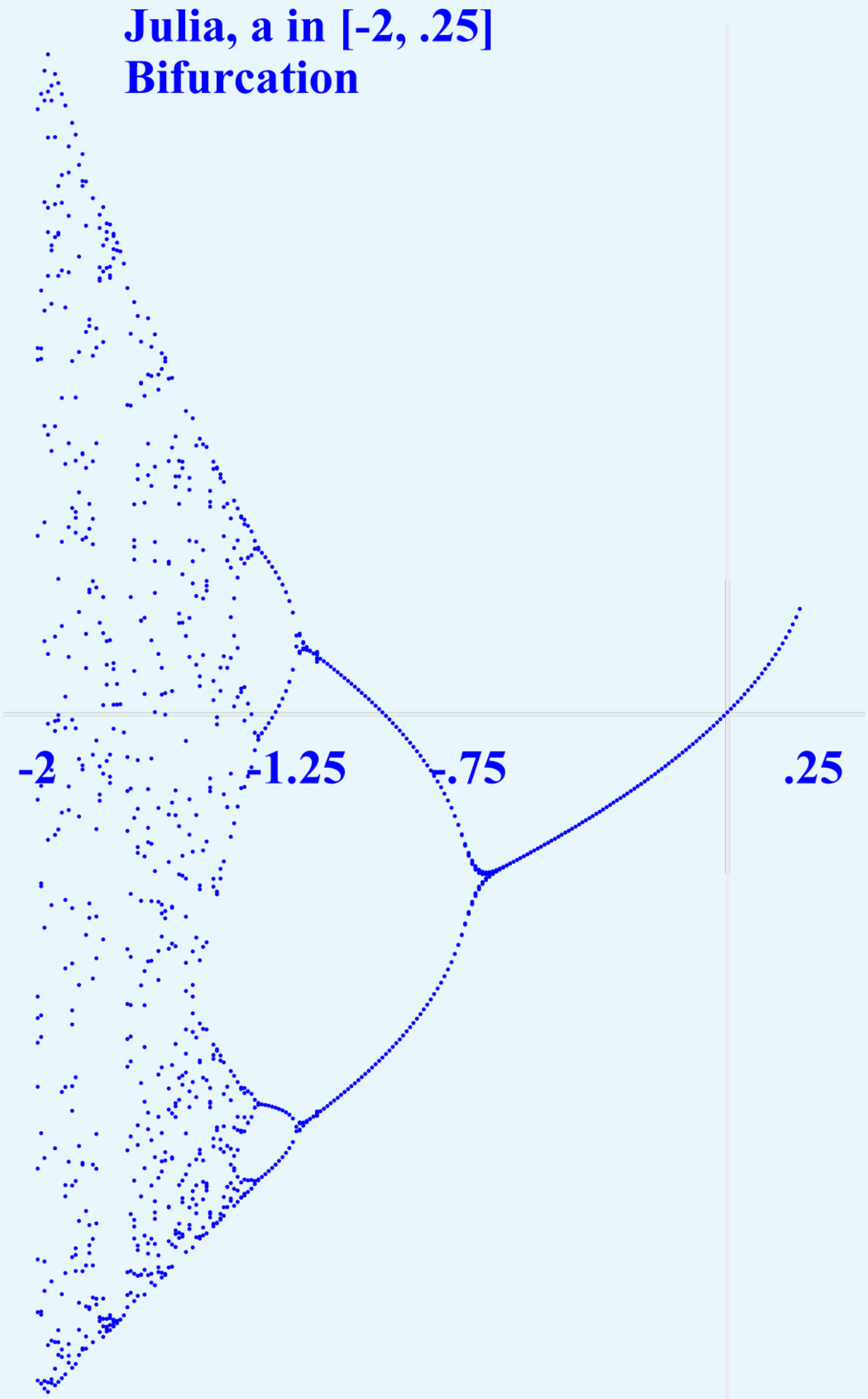
Stability

$|f'(l_1)| = |1 + \sqrt{5}| > 1$ **repeller**

$|f'(l_2)| = |1 - \sqrt{5}| > 1$ **repeller**

Julia fractal bifurcation diagram

Julia fractal bifurcation diagram



Julia fractal J(-1)

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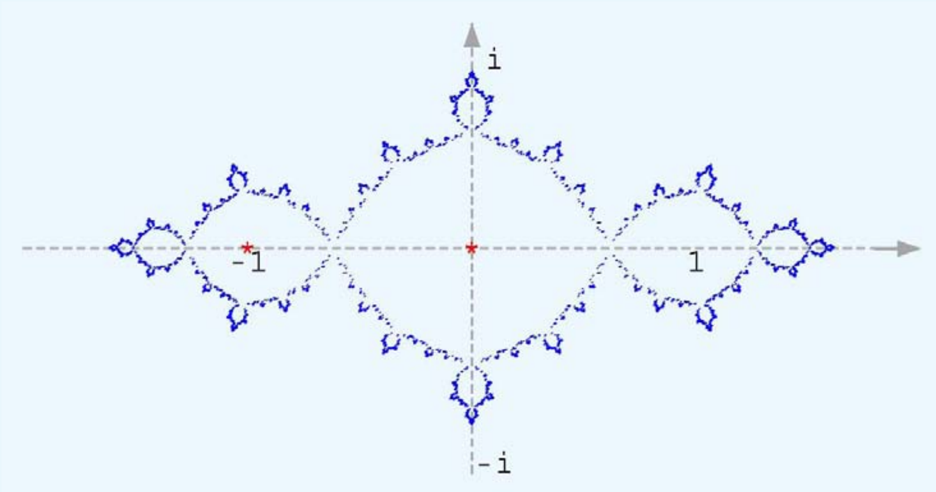
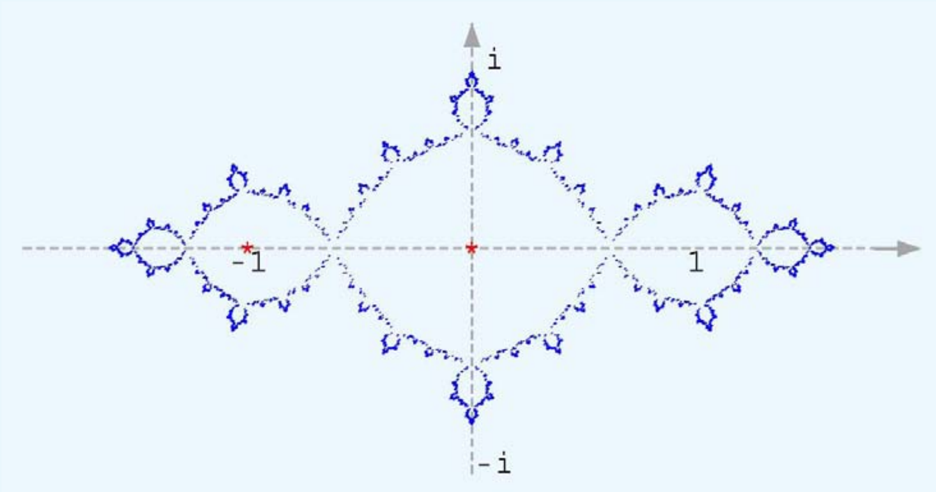


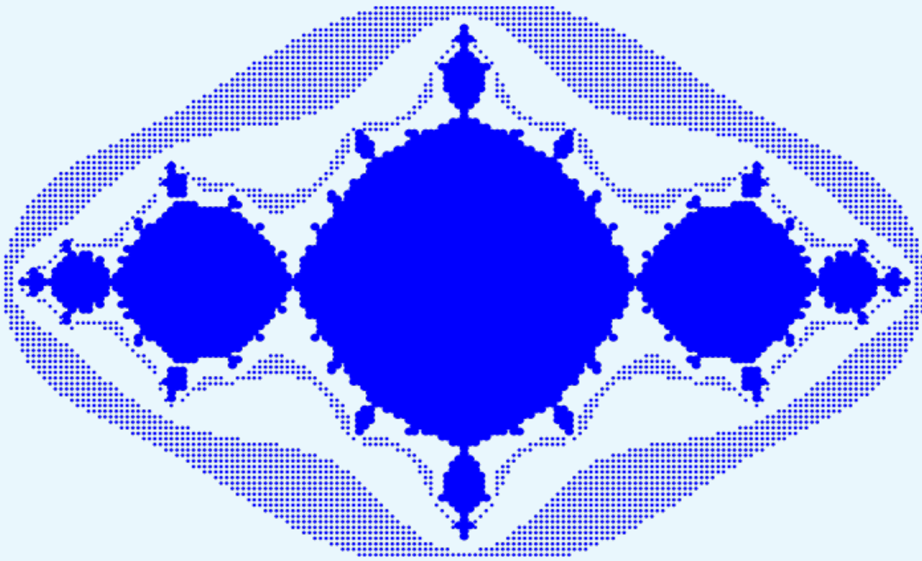
Julia fractal J(-1)



Points within Julia fractal
don't fly away to infinity
They either stay on the fractal
or converge to
strange attractors 0 and -1

Julia fractal J(-1)

points fly away to infinity



Julia fractal programs

Julia fractal programs Lauwerier

JULIAMC

Julia fractal programsLauwerier

JULIAMC
JULIABS

Julia fractal programs Lauwerier

JULIAMC
JULIABS
JULIAF

Julia fractal programs$_{\text{Lauwerier}}$

JULIAMC
JULIABS
JULIAF
JULIADistance
JULIADETail

. . .

JULIAMC Inverse Iteration

JULIAMC Inverse Iteration

$$z_{i-1} = \pm\sqrt{z_i - c} \qquad i = n + 10, \ldots, 1$$

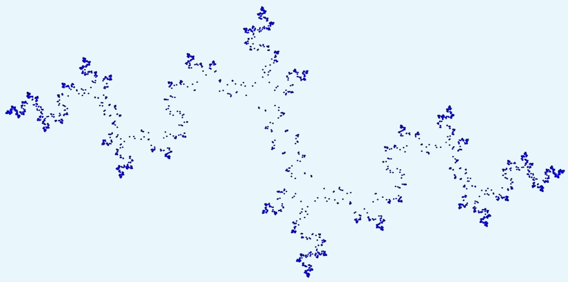
JULIAMC Inverse Iteration

$$z_{i-1} = \pm\sqrt{z_i - c} \qquad i = n+10, \ldots, 1$$

```
%!PS-Adobe-3.0 EPSF-3.0
%%BoundingBox: -165 -85 165 85
%%BeginSetup
%%EndSetup
%%BeginProlog
(C:\\PSlib\\PSlib.eps) run%PSlib
%%EndProlog
%
% Program ---the script---
%
-1 0 5000 JULIAMC       %SanMarco
showpage
%%EOF
```

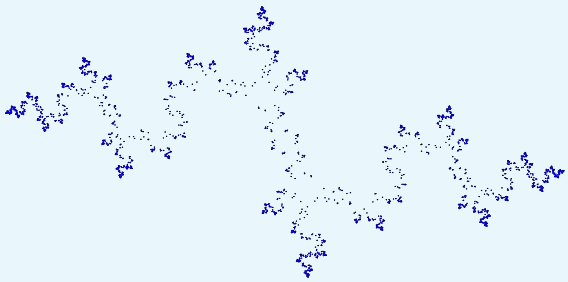
Examples I

Examples I



-1.03 .386 5000 JULIAMC

Examples I



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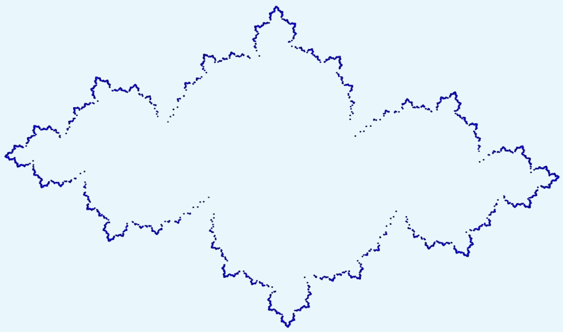
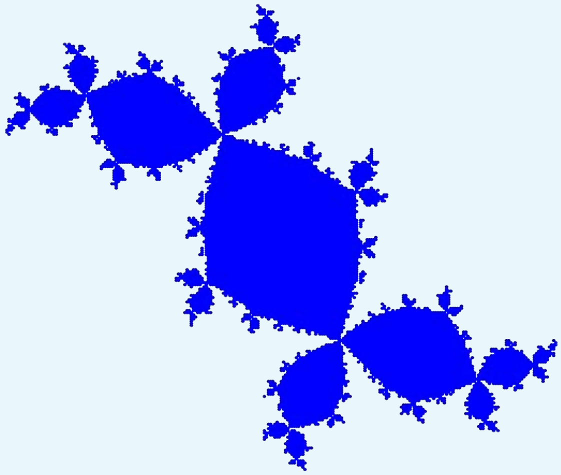


-.8 .15 5000 JULIAMC

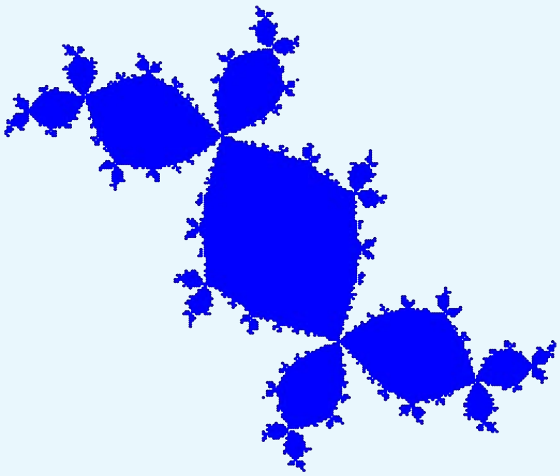
Examples II

Examples II

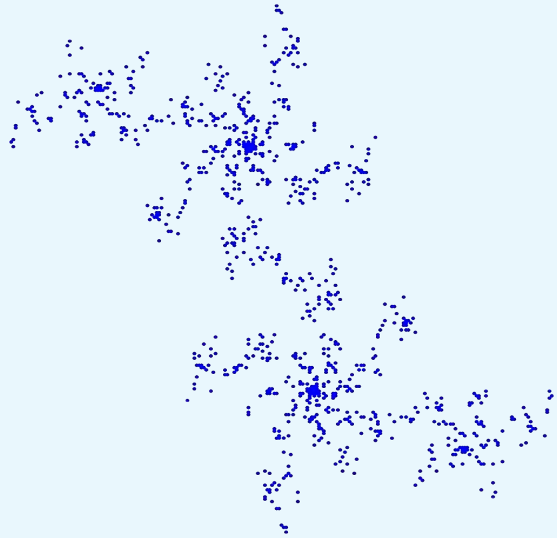


-.12 .74 1.2 1.3 80 JULIAF

Examples II



-.12 .74 1.2 1.3 80 JULIAF



.11 .66 2.1 1.85 80 JULIAF

JULIADistance

distance formula

$$d(z_0, J) \approx |z_n| \log |z_n| \, / \, |\frac{dz_n}{dz}|$$

JULIADistance

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$$\mathrm{d}(z_0, J) \approx |z_n| \log |z_n| \,/\, |\frac{dz_n}{dz}|$$

gives more details

Examples III

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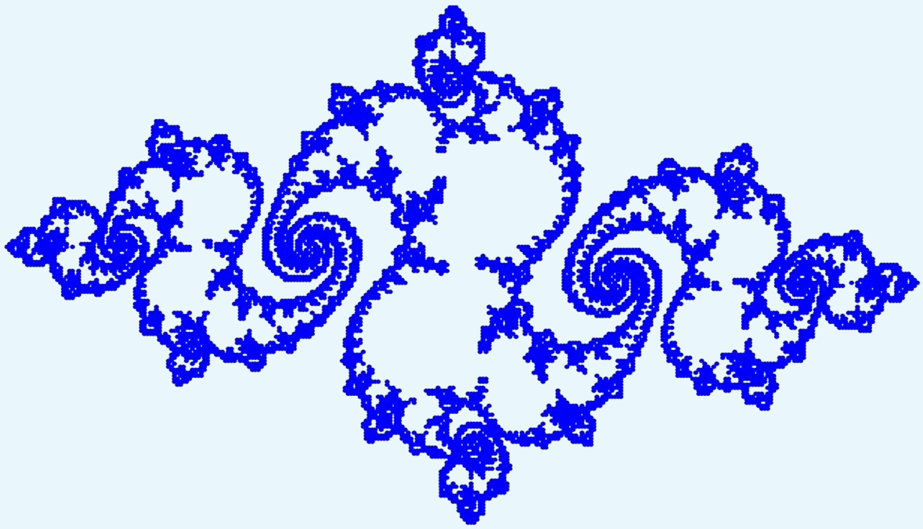
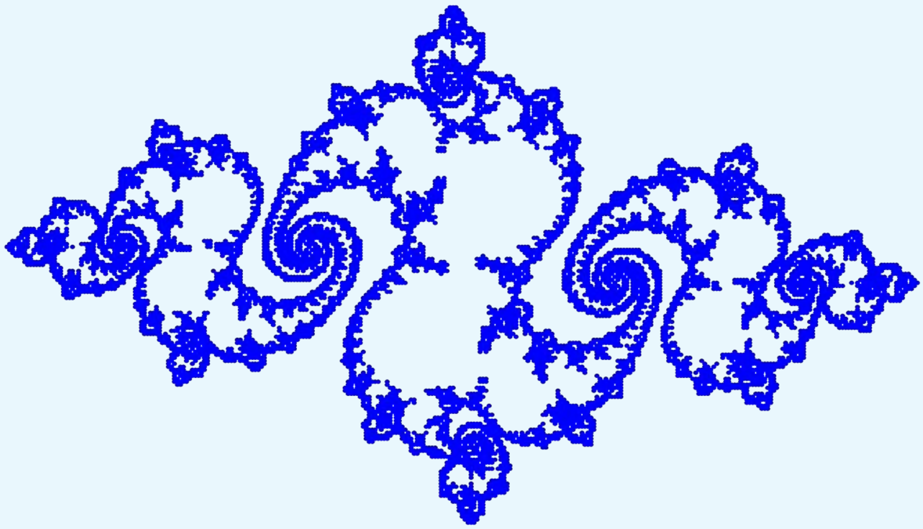


-.775 .1103 1.6 .9 125 .01 JULIAD

Examples III



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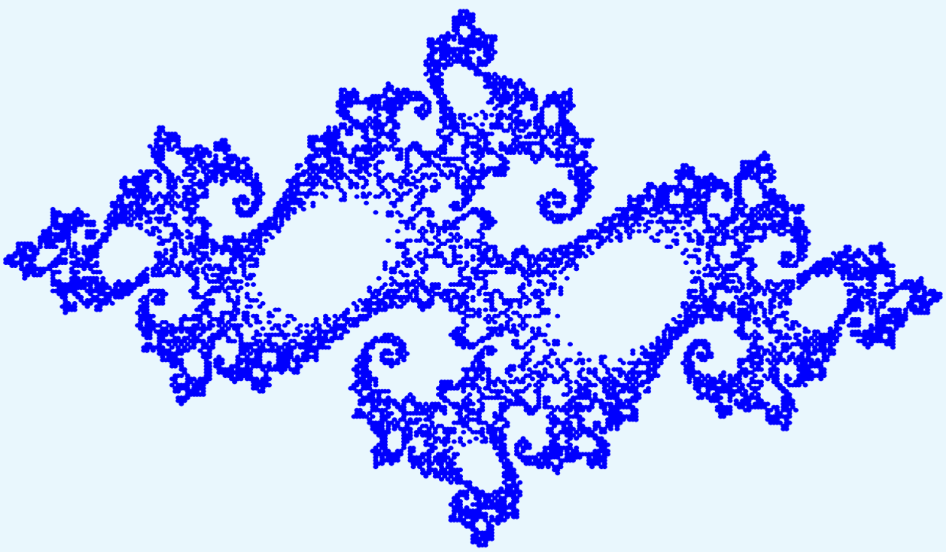
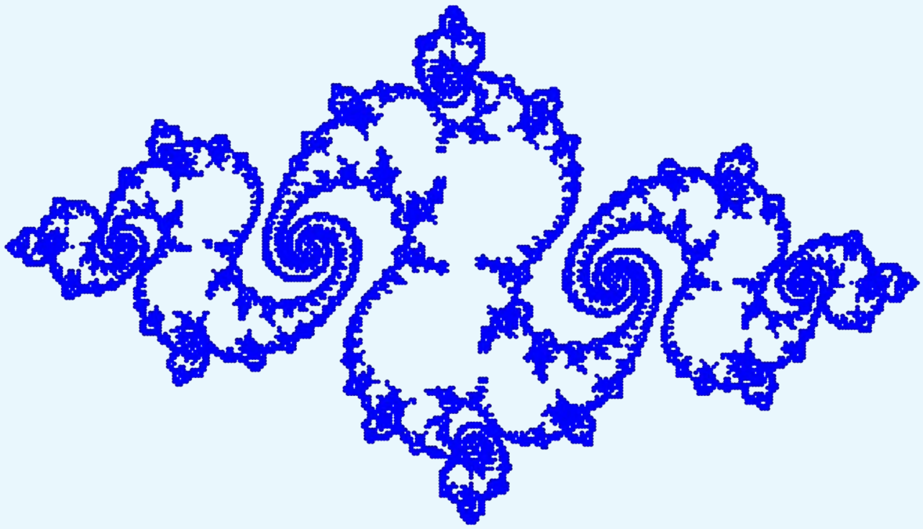
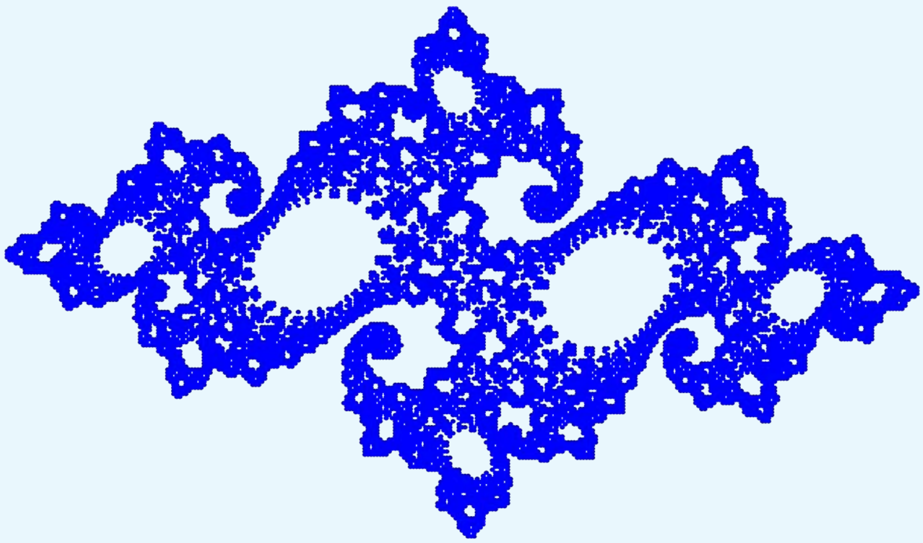


-.7454 .1103 1.5 100 .001 JULIAD

Examples III

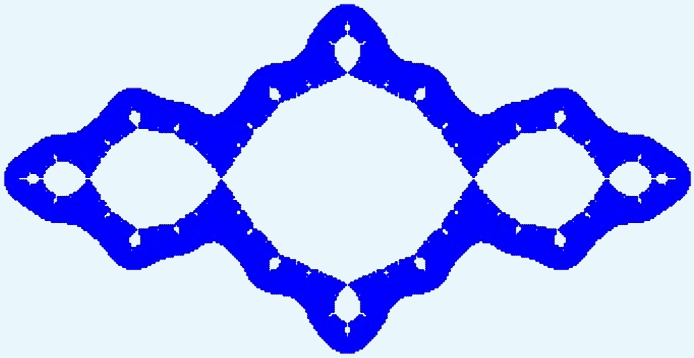


-.775 .1103 1.6 .9 125 .01 JULIAD

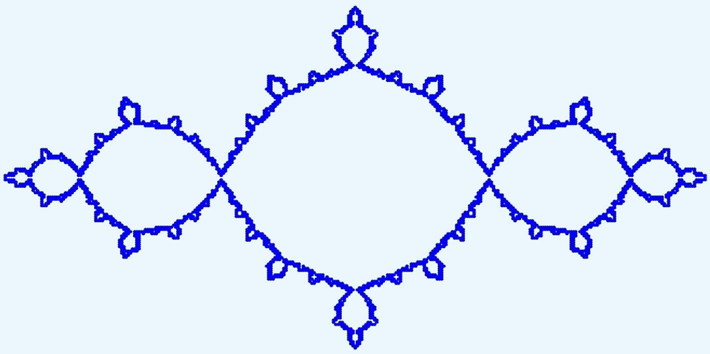


-.7454 .1103 1.6 .9 125 .01 JULIAD

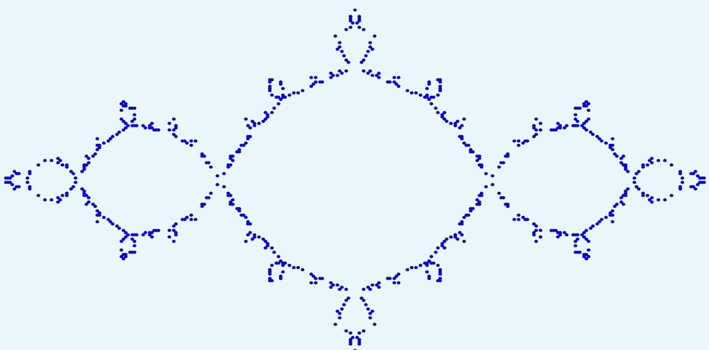
Examples IV



-1 0 1.65 .85 50 **0.1** JULIAD

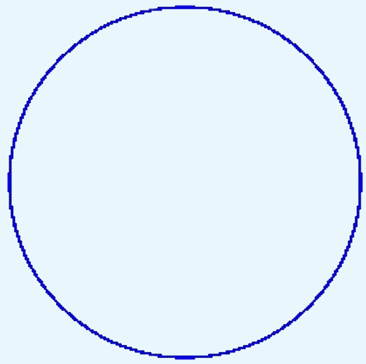


-1 0 1.65 .85 50 **.01** JULIAD



-1 0 1.65 .85 50 **.001** JULIAD

Examples V

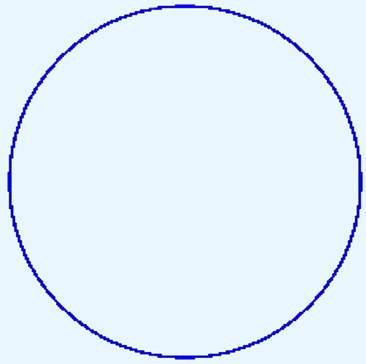


0 0 1.1 1.1 50 0.1 JULIAD

-2 0 2 .1 50 .01 JULIAD

$$\mathrm{J}(0) \xrightarrow{z+\frac{1}{z}} \mathrm{J}(\text{-}2)$$

Examples V



0 0 1.1 1.1 50 0.1 JULIAD

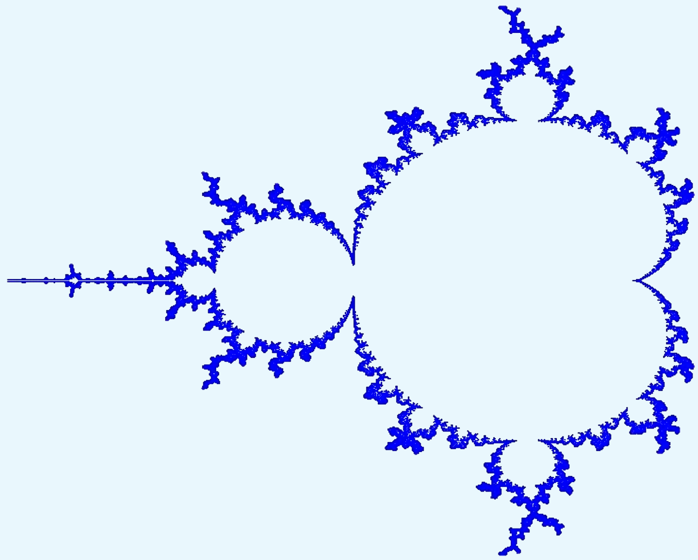
-2 0 2 .1 50 .01 JULIAD

$$J(0) \xrightarrow{z+\frac{1}{z}} J(\text{-}2)$$

-3.45 0 2 .1 50 .0000001 JULIAD

Mandelbrot

Mandelbrot
Map of connected J-fractals



Zooming-in

Zooming-in

MANDET & MANDIS

MANDET & MANDIS

distance formula

$$\mathrm{d}(c, M) \approx |p_n| \log |p_n| \,/\, |\frac{dp_n}{dc}|$$

gives more details

MANDET

```
-1.927199 0 .0005 100 MANDET
```

MANDET

```
-1.927199 0 .0005 100 MANDET
```

```
-1.749057 0.000306 .04 100 MANDET
```

Cont...

```
-1.25636 0.38032 .08 100 MANDET
```

Cont. . .

```
-1.25636 0.38032 .08 100 MANDET
```

```
-0.7489 0.1073 0.004 100 MANDET
```

Examples V

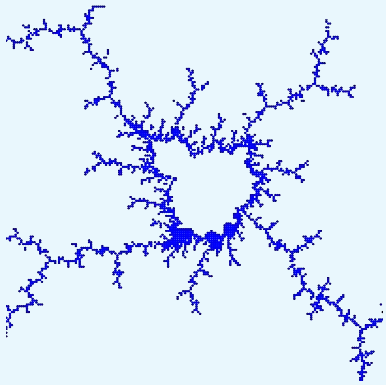
-.7454 0 1.5 100 MANDET

-.7454 0 1 100 MANDET

-.7454 0 .5 100 MANDET

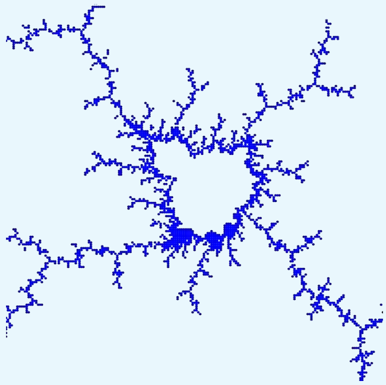
MANDIS

MANDIS

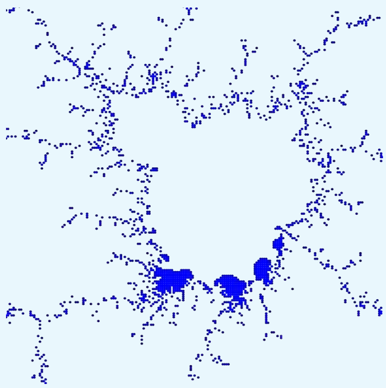


-.16 1.03 .025 50 .00005 MANDISm

MANDIS



-.16 1.03 .025 50 .00005 MANDISm



-.16 1.03 .0125 50 .000005 MANDISm

So far serious matter

let's go over to

where you've been waiting for

So far serious matter

let's go over to

where you've been waiting for

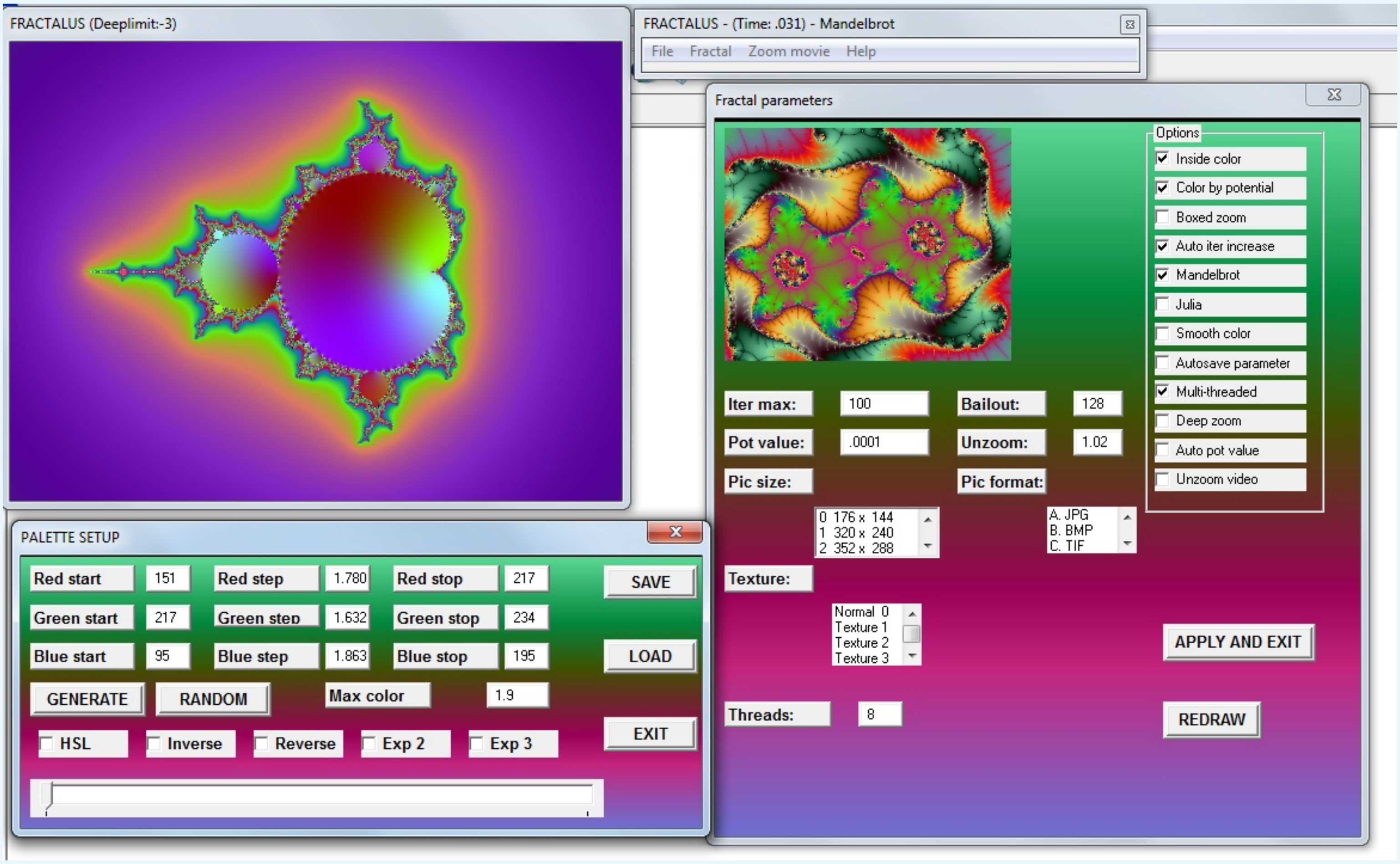
FUN

Packages from WWW

Packages from WWW

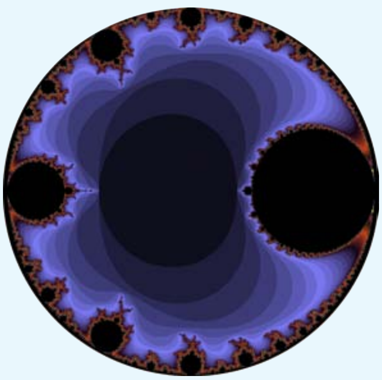
. . . not only should we show
in our User Group publications
what TEX&Co can do for us
but also mention programs
which perform the same task . . .

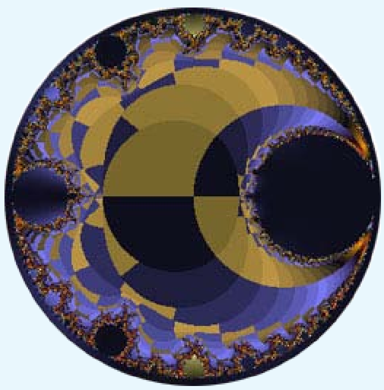
XaoS, Fractulus, ...



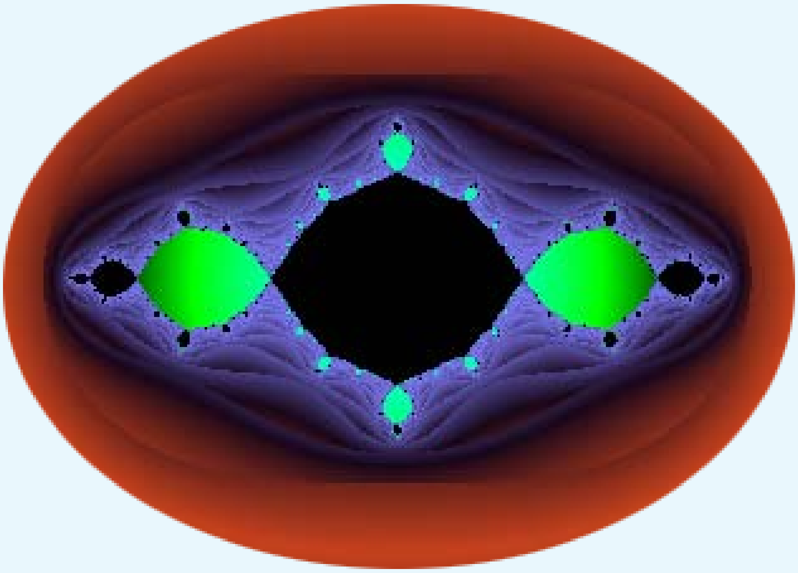
XaoS

XaoS





Result user formula $z^2 + \{-.8, 0\}$



J(-.8, 0) San Marco

Conclusions

Conclusions
- PostScript defs in PSlib.eps

Conclusions

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- M-fractal map Julia fractals

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- M-fractal bifurcation diagram

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- no fractal contours
- (pdf)TeX PostScript remarks

XaoS movie VIII

plain $\TeX$ remarks

plain TEX remarks

- too large 10^{-8} in footnotes
  too small here

plain TeX remarks

- too large 10^{-8} in footnotes
  too small here
- in general:
  Fonts not context-sensitive
  unless...

(pdf)TEX remarks

Insert needed of

- `\pdfliteral{1 0 0 0 k}` → **blue TEXt**

as well as

- `\pdfliteral{1 0 0 0 K}` → **blue lines**

(pdf)TeX remarks

Insert needed of

- `\pdfliteral{1 0 0 0 k}` $\rightarrow$ **blue TeXt**

as well as

- `\pdfliteral{1 0 0 0 K}` $\rightarrow$ **blue lines**

$$\sqrt{} \quad \leftrightarrow \quad \sqrt{}$$

Error-prone

(pdf)TeX remarks

Insert needed of

- `\pdfliteral{1 0 0 0 k}` $\rightarrow$ **blue TeXt**

as well as

- `\pdfliteral{1 0 0 0 K}` $\rightarrow$ **blue lines**

$$\sqrt{} \quad \leftrightarrow \quad \sqrt{}$$

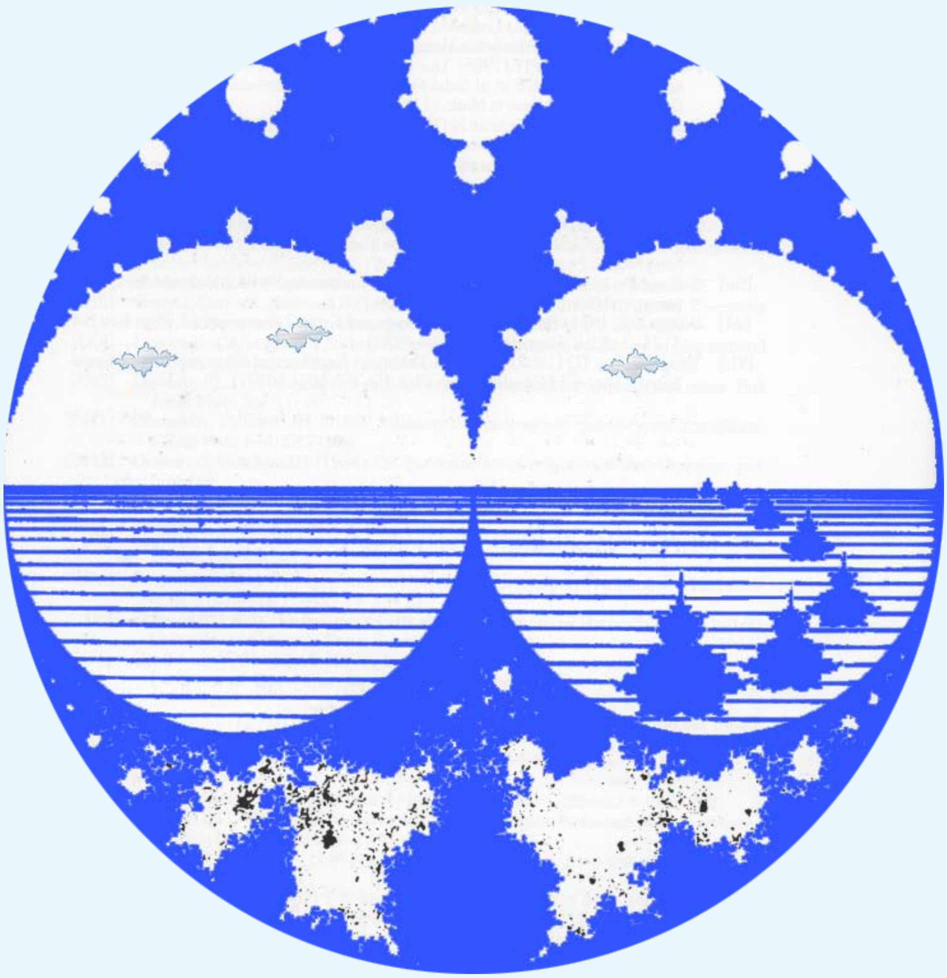
Error-prone

- **pdfTeX, OTF step forwards but** . . .

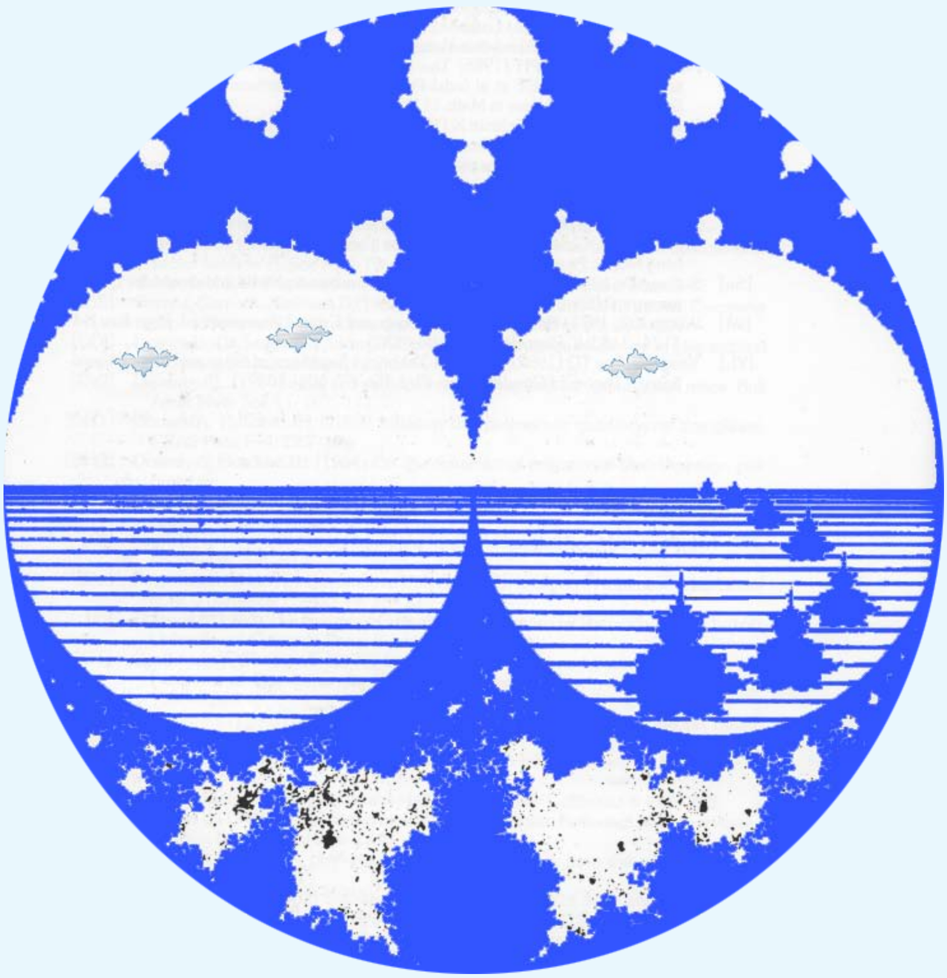
  `\psfig` **functionality lost**

Mandelbrot's view of Breskens

Mandelbrot's view of Breskens



Mandelbrot's view of Breskens



Thank you, Bye!