Using LaTeX as a computing language

Introduction

It is possible to use LaTeX as a primitive computing language to do some repetitive calculations, examples of which are

- calculation of dates for a timetable
- vector and scalar products
- solution of second-order constant coefficient linear differential equations
- cancelling common factors in a fraction
- adding up marks on a student's script

My question is whether these are useful? If so, whether it is worth extending this?

Dates on a timetable

Introduction. I produce a timetable for the lecture course I give. The dates on the timetable are a constant number of days relative to the start date (e.g. if the start date is 8 January, then the first computing practical is on 22 January, a fortnight later).

I had done my own programming before I was aware of the `datenumber` and `ifthen` packages, and would use those instead now.

The material is used for a slide at the beginning of the course, and in LaTeX 2.09 format—don't change a winning formula.

Coding. The coding is within the actual input file, and not a separate package.

```
\makeatletter
   \newcount{\@startdate}
   \@startdate=0
   \newcount{\@tempdate}
   \newcount{\@dayplus}
   \newcommand{\startdate}[1]%
      {\advance \@startdate by #1}%
   \newcommand{\dayplus}[1]%
      {\@tempdate = \@startdate \advance \@tempdate by #1%
      \ifnum \@tempdate < 32 \the\@tempdate~January%
         \else \advance \@tempdate by -31 \the\@tempdate~February\fi}%
\makeatother

\begin{slide}{}
   \begin{center} \bf Format of this component\end{center}
   \begin{description}
      \item[7 Formal lectures] on Tuesdays and Thursdays
         {\em start at 9 o'clock}
         \startdate{15}
```

```
      \item[4 Informal Lectures] on Saturdays,
         including this Saturday, \dayplus{2}
      \item[Examples classes] on Thursday \dayplus{7}
         in Computing Room, 2.00 -- 3.15\,p.m., \\3.30 -- 4.45\,p.m.,
         4.45 -- 6.00\,p.m.
      \item[Computer] Practical Classes  on
         Thursdays\\ \dayplus{14}  and \dayplus{21},
         2.00 -- 3.15\,p.m., 3.30 -- 4.45\,p.m., 4.45 -- 6.00\,p.m.
      \item[Assessed practical]
         Thursday \dayplus{28}, \\ 2.00 -- 3.15\,p.m.,
         3.30 -- 4.45\,p.m., \\4.45 -- 6.00\,p.m.
   \end{description}
\end{slide}
```

Example of output. This produces the output in Figure 1.

Format of this component

7 Formal lectures on Tuesdays and Thursdays *start at 9 o'clock*

4 Informal Lectures on Saturdays, including this Saturday, 14 January

Examples classes on Thursday 19 January in Computing Room, 2.00 – 3.15 p.m., 3.30 – 4.45 p.m., 4.45 – 6.00 p.m.

Computer Practical Classes on Thursdays 26 January and 2 February, 2.00 – 3.15 p.m., 3.30 – 4.45 p.m., 4.45 – 6.00 p.m.

Assessed practical Thursday 9 February, 2.00 – 3.15 p.m., 3.30 – 4.45 p.m., 4.45 – 6.00 p.m.

1

Figure 1. Example of dates on a timetable

Calculating cross- and dot-products of vectors

Definitions. Exercises involving the products, be they cross or dot, of vectors are invariably with integer coefficients, and so it is easy to calculate them using integer arithmetic.

If two vectors $\mathbf{a}$ and $\mathbf{b}$ have Cartesian components a_1, a_2, a_3 (or $a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$) and b_1, b_2, b_3 (or $b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$) respectively, then the dot (or scalar) product is defined[1] by

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

and the cross (or vector) product is defined by

$$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}.$$

Although it is easy to do the arithmetic using the `calc` package, good style in mathematics dictates that $0\mathbf{i} + 4\mathbf{j} + 0\mathbf{k}$ is not as elegant as $4\mathbf{j}$ and that intermediate calculations that involve a negative number ought to have that negative number in brackets, e.g. $4 \times -3 - -3 \times 2$ should be replaced by $(4 \times (-3)) - ((-3) \times 2)$.

So in this package, I have tried to avoid these lapses of style.

Coding of the package. The package coding is:

```
\RequirePackage{calc, ifthen}

\renewcommand{\i}{{\mathbf i}}
\renewcommand{\j}{{\mathbf j}}
\renewcommand{\k}{{\mathbf k}}

\newcommand{\bfcdot}%
    {\mathbin{\mbox{\normalfont \bf \raise 0.4ex \hbox{.}}}}

\newcommand{\bftimes}{\mathbin{\mbox{\normalfont \bf\textsf{x}}}}

%% Define the counters for each of the components and a temporary one.
\newcounter{@ai}\newcounter{@aj}\newcounter{@ak}
\newcounter{@bi}\newcounter{@bj}\newcounter{@bk}
\newcounter{@ci}\newcounter{@cj}\newcounter{@ck}
\newcounter{@temp}

%% A command to include brackets if the value is less than zero
\newcommand{\@negbrackets}[1]%
    {\setcounter{@temp}{#1}
    \ifthenelse{\value{@temp}<0}%
        {(\the@temp)}%
        {\the@temp}
    } % end \@negbrackets

%% to check whether a previous component exists
\newboolean{@previous}
\setboolean{@previous}{false}

%% to be used for the first non-zero component
\newcommand{\@firstcomponent}[2]%
    {\setcounter{@temp}{#1}
    \ifthenelse{\value{@temp}=0}%
        {\relax}%
        {\setboolean{@previous}{true}
        \ifthenelse{\value{@temp}=1}%
            {#2}%
            {\the@temp\,#2}
        }
    } % end \@firstcomponent

%% to be used if the previous component is not zero
\newcommand{\@subsequentcomponent}[2]%
    {\setcounter{@temp}{#1}
    \ifthenelse{\boolean{@previous}}%
        {\ifthenelse{\value{@temp}=0}%
            {\relax}%
            {\ifthenelse{\value{@temp}=1}%
                {+#2}%
                {\ifthenelse{\value{@temp}=-1}%
                    {-#2}%
                    {\ifthenelse{\value{@temp}<0}%
                        {\the@temp\,#2}%
                        {+\the@temp\,#2}
                    }
                }
            }
```

```
        }%
        {\@firstcomponent{#1}{#2}}%
    } % end \@subsequentcomponent

%% writes a zero vector if all previous components are zero,
%% and resets the switch
\newcommand{\@zerovector}%
    {\ifthenelse{\boolean{@previous}}%
        {\setboolean{@previous}{false}}%
        {\mathbf 0}
    } % ends \@zerovector

%% puts it all together and produces the output
\newcommand{\crossproduct}[6]%
    {\setcounter{@ai}{#1}\setcounter{@aj}{#2}\setcounter{@ak}{#3}
    \setcounter{@bi}{#4}\setcounter{@bj}{#5}\setcounter{@bk}{#6}
    \setcounter{@ci}{\value{@aj}*\value{@bk}-\value{@ak}*\value{@bj}}
    \setcounter{@cj}{\value{@ak}*\value{@bi}-\value{@ai}*\value{@bk}}
    \setcounter{@ck}{\value{@ai}*\value{@bj}-\value{@aj}*\value{@bi}}%
    (\@firstcomponent{#1}{\i}%
     \@subsequentcomponent{#2}{\j}\@subsequentcomponent{#3}{\k}
        \@zerovector) \bftimes
    (\@firstcomponent{#4}{\i}%
     \@subsequentcomponent{#5}{\j}\@subsequentcomponent{#6}{\k}
        \@zerovector)\\
    &=& \left(\@negbrackets{\the@aj}\times\@negbrackets{\the@bk}
        -\@negbrackets{\the@ak}\times\@negbrackets{\the@bj} \right)\i +
        \left(\@negbrackets{\the@ak}\times\@negbrackets{\the@bi}
        -\@negbrackets{\the@ai}\times\@negbrackets{\the@bk}\right)\j +
        \left(\@negbrackets{\the@ai}\times\@negbrackets{\the@bj}
        -\@negbrackets{\the@aj}\times\@negbrackets{\the@bi}\right)\k \\
    &=& \@firstcomponent{\the@ci}{\i}   \@subsequentcomponent{\the@cj}{\j}
          \@subsequentcomponent{\the@ck}{\k}\@zerovector
    } %ends \crossproduct

\newcommand{\dotproduct}[6]%
    {\setcounter{@ai}{#1}\setcounter{@aj}{#2}\setcounter{@ak}{#3}
    \setcounter{@bi}{#4}\setcounter{@bj}{#5}\setcounter{@bk}{#6}
    \setcounter{@ci}{\value{@ai}*\value{@bi}+\value{@aj}*\value{@bj}+
    \value{@ak}*\value{@bk}}%
    (\@firstcomponent{#1}{\i}%
     \@subsequentcomponent{#2}{\j}%
     \@subsequentcomponent{#3}{\k}
        \@zerovector) .
    (\@firstcomponent{#4}{\i}%
     \@subsequentcomponent{#5}{\j}\@subsequentcomponent{#6}{\k}
        \@zerovector)\\
    &=& \@negbrackets{\the@ai}\times\@negbrackets{\the@bi}
        +\@negbrackets{\the@aj}\times\@negbrackets{\the@bj}
        +\@negbrackets{\the@ak}\times\@negbrackets{\the@bk} \\
    &=& \the@ci
    } %ends \dotproduct
```

Input to the package. With this package, the following produces the output in Figure 2.

```
\begin{eqnarray*}
    \mathbf{p} \bftimes \mathbf{q}& = &\crossproduct{-3}{-1}{6}{2}{-2}{4}
\end{eqnarray*}

\begin{eqnarray*}
```

```
    \mathbf{p} \bftimes \mathbf{q}& = &\crossproduct{8}{-1}{6}{0}{0}{4}
\end{eqnarray*}

\begin{eqnarray*}
    \mathbf{q}\bftimes \mathbf{r} &=& \crossproduct{-3}{0}{4}{0}{0}{4}
\end{eqnarray*}

\begin{eqnarray*}
    \mathbf{q}\bftimes \mathbf{r} &=& \crossproduct{0}{0}{0}{0}{0}{0}
\end{eqnarray*}

\begin{eqnarray*}
    \mathbf{p} \bfcdot \mathbf{q}& = &\dotproduct{-3}{-1}{6}{2}{-2}{4}
\end{eqnarray*}

\begin{eqnarray*}
    \mathbf{p} \bfcdot \mathbf{q}& = &\dotproduct{8}{-1}{6}{0}{0}{4}
\end{eqnarray*}

\begin{eqnarray*}
    \mathbf{q}\bfcdot \mathbf{r} &=& \dotproduct{-3}{0}{4}{0}{0}{4}
\end{eqnarray*}

\begin{eqnarray*}
    \mathbf{q}\bfcdot \mathbf{r} &=& \dotproduct{0}{0}{0}{0}{0}{0}
\end{eqnarray*}
```

A very similar package is written for calculating determinants of 2×2 and 3×3 matrices.

Solution of second-order constant coefficient homogeneous linear differential equations

Definitions. I shan't be solving differential equations; but *reverse engineer* them from the solution.

If the solutions to the auxiliary equation are real, say λ_1 and λ_2, then the differential equation is

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - (\lambda_1 + \lambda_2)\frac{\mathrm{d}y}{\mathrm{d}x} + \lambda_1\lambda_2 y = 0$$

and has solution

$$y = A\mathrm{e}^{\lambda_1 x} + B\mathrm{e}^{\lambda_2 x}.$$

So first choose λ_1 and λ_2 and then generate the question and solution from those two numbers.

If the solutions to the auxiliary equation are complex, say $\alpha \pm \beta\mathrm{i}$, then the differential equation is

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\alpha\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\alpha^2 + \beta^2\right) y = 0$$

and has solution

$$y = \mathrm{e}^{\alpha x}\left(A\cos(\beta x) + B\sin(\beta x)\right).$$

So first choose α and β and generate the question and solution from those two numbers.

Coding.

```
\RequirePackage{ifthen, calc}
\parindent 0pt

\newboolean{@positive} % to cater for negative sign
```

$$
\begin{aligned}
\mathbf{p}\times\mathbf{q} &= (-3\,\mathbf{i}-\mathbf{j}+6\,\mathbf{k})\times(2\,\mathbf{i}-2\,\mathbf{j}+4\,\mathbf{k})\\
&= ((-1)\times 4-6\times(-2))\,\mathbf{i}+(6\times 2-(-3)\times 4)\,\mathbf{j}+((-3)\times(-2)-(-1)\times 2)\,\mathbf{k}\\
&= 8\,\mathbf{i}+24\,\mathbf{j}+8\,\mathbf{k}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{p}\times\mathbf{q} &= (8\,\mathbf{i}-\mathbf{j}+6\,\mathbf{k})\times(4\,\mathbf{k})\\
&= ((-1)\times 4-6\times 0)\,\mathbf{i}+(6\times 0-8\times 4)\,\mathbf{j}+(8\times 0-(-1)\times 0)\,\mathbf{k}\\
&= -4\,\mathbf{i}-32\,\mathbf{j}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{q}\times\mathbf{r} &= (-3\,\mathbf{i}+4\,\mathbf{k})\times(4\,\mathbf{k})\\
&= (0\times 4-4\times 0)\,\mathbf{i}+(4\times 0-(-3)\times 4)\,\mathbf{j}+((-3)\times 0-0\times 0)\,\mathbf{k}\\
&= 12\,\mathbf{j}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{q}\times\mathbf{r} &= (\mathbf{0})\times(\mathbf{0})\\
&= (0\times 0-0\times 0)\,\mathbf{i}+(0\times 0-0\times 0)\,\mathbf{j}+(0\times 0-0\times 0)\,\mathbf{k}\\
&= \mathbf{0}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{p}\cdot\mathbf{q} &= (-3\,\mathbf{i}-\mathbf{j}+6\,\mathbf{k}).(2\,\mathbf{i}-2\,\mathbf{j}+4\,\mathbf{k})\\
&= (-3)\times 2+(-1)\times(-2)+6\times 4\\
&= 20
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{p}\cdot\mathbf{q} &= (8\,\mathbf{i}-\mathbf{j}+6\,\mathbf{k}).(4\,\mathbf{k})\\
&= 8\times 0+(-1)\times 0+6\times 4\\
&= 24
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{q}\cdot\mathbf{r} &= (-3\,\mathbf{i}+4\,\mathbf{k}).(4\,\mathbf{k})\\
&= (-3)\times 0+0\times 0+4\times 4\\
&= 16
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{q}\cdot\mathbf{r} &= (\mathbf{0}).(\mathbf{0})\\
&= 0\times 0+0\times 0+0\times 0\\
&= 0
\end{aligned}
$$

Figure 2. Example of dot and cross products

```
\setboolean{@positive}{true}

\newcounter{@a}\newcounter{@b}
\newcounter{@trace}\newcounter{@det}
\newcounter{diffcoefficient} % for external use
\newcounter{functioncoefficient} % for external use
\newcounter{@x}
\setcounter{@a}{3}
```

```
\setcounter{@b}{4}

\newcommand{\sign}{\ifthenelse{\boolean{@positive}}{+}{-}}

\newcommand{\dequreal}[2]%
        {\setcounter{@a}{#1}\setcounter{@b}{#2}
        \setcounter{@x}{\value{@a}+\value{@b}}
        \setcounter{diffcoefficient}{-\value{@a}-\value{@b}}
        \setcounter{functioncoefficient}{\value{@a}*\value{@b}}
        \frac{\mathrm{d}^2y}{\mathrm{d}x^2}
        \ifthenelse{\value{@x}=0}%
            {\relax}%
            {\ifthenelse{\value{@x}<0}%
                {\setcounter{@trace}{-\value{@x}}\setboolean{@positive}{true}}%
                {\setboolean{@positive}{false}\setcounter{@trace}{\value{@x}}}
            \sign
            \ifthenelse{\value{@trace}=1}%
                {\relax}%
                {\the@trace} %% to remove 1
            \frac{\mathrm{d}y}{\mathrm{d}x}
        }
        \setcounter{@x}{{\value{@a}*\value{@b}}}
        \ifthenelse{\value{@x}<0}%
            {\setcounter{@det}{-\value{@x}}\setboolean{@positive}{false}}%
            {\setcounter{@det}{\value{@x}}\setboolean{@positive}{true}}
        \sign
        \ifthenelse{\value{@det}=1}%
            {\relax}%
            {\the@det} %% to remove 1
        y=0 %\quad (\mbox{arguments are } #1,#2)
    } % ends dequreal

\newcommand{\auxiliaryreal}[2]%
        {\setcounter{@a}{#1}\setcounter{@b}{#2}
        \setcounter{@x}{\value{@a}+\value{@b}}
        \lambda^2
        \ifthenelse{\value{@x}=0}%
            {\relax}%
            {\ifthenelse{\value{@x}<0}%
                {\setcounter{@trace}{-\value{@x}}\setboolean{@positive}{true}}%
                {\setboolean{@positive}{false}\setcounter{@trace}{\value{@x}}}
            \sign
            \ifthenelse{\value{@trace}=1}{\relax}{\the@trace} %% to remove 1
            \lambda}
        \setcounter{@x}{{\value{@a}*\value{@b}}}
        \ifthenelse{\value{@x}<0}%
            {\setcounter{@det}{-\value{@x}}\setboolean{@positive}{false}}%
            {\setcounter{@det}{\value{@x}}\setboolean{@positive}{true}}
        \sign   \ifthenelse{\value{@det}=1}{\relax}{\the@det} %% to remove 1
        =0
    } % ends \auxiliaryreal

\newcommand{\auxiliaryfactors}[2]%
    {(\lambda \setcounter{@x}{-#1}
    \ifthenelse{\value{@x}<0}%
        {\relax}%
        {+}
    \the@x)
    (\lambda \setcounter{@x}{-#2}
    \ifthenelse{\value{@x}<0}%
```

```
        {\relax}%
        {+}
    \the@x) =0
    } % ends \auxiliaryfactors

\newcommand{\cfreal}[2]%
    {y=A\mathrm{e}^{#1 x}+B\mathrm{e}^{#2 x}
    } % ends \cfreal

\newcommand{\dequcomplex}[2]%
        {\setcounter{@a}{#1}\setcounter{@b}{#2}
        \setcounter{@x}{2*\value{@a}}
        \setcounter{diffcoefficient}{-2*\value{@a}}
        \setcounter{functioncoefficient}
                        {{\value{@a}*\value{@a}+\value{@b}*\value{@b}}}
        \frac{\mathrm{d}^2y}{\mathrm{d}x^2}
        \ifthenelse{\value{@x}=0}%
            {\relax}%
            {\ifthenelse{\value{@x}<0}%
                {\setcounter{@trace}{-\value{@x}}\setboolean{@positive}{true}}%
                {\setboolean{@positive}{false}\setcounter{@trace}{\value{@x}}}
            \sign
            \ifthenelse{\value{@trace}=1}{\relax}{\the@trace} %% to remove 1
        \frac{\mathrm{d}y}{\mathrm{d}x}}
        \setcounter{@x}{{\value{@a}*\value{@a}+\value{@b}*\value{@b}}}
        + \the@x    y=0 %\quad (\mbox{arguments are } #1,#2)
    } % ends \dequcomplex

\newcommand{\auxiliarycomplex}[2]%
    {\setcounter{@a}{#1}\setcounter{@b}{#2}
    \setcounter{@x}{2*\value{@a}}
    \lambda^2
    \ifthenelse{\value{@x}=0}%
        {\relax}%
        {\ifthenelse{\value{@x}<0}%
            {\setcounter{@trace}{-\value{@x}}\setboolean{@positive}{true}}%
            {\setboolean{@positive}{false}\setcounter{@trace}{\value{@x}}}
        \sign
        \ifthenelse{\value{@trace}=1}%
            {\relax}%
            {\the@trace} %% to remove 1
        \lambda
        }
    \setcounter{@x}{{\value{@a}*\value{@a}+\value{@b}*\value{@b}}}
    + \the@x y=0
    } % ends \auxiliarycomplex

\newcommand{\bsquared}{\relax}

\newcommand{\bsquaredd}[1]{\renewcommand{\bsquared}{#1}}

\newcommand{\auxiliaryquadratic}[2]%
    {\setcounter{@a}{#1}\setcounter{@b}{#2}
    \ifthenelse{\value{@b}<0}%
        {\setcounter{@b}{-\value{@b}}}%
        {\relax}
    \setcounter{@x}{2*\value{@a}}
    \ifthenelse{\value{@a}>0}%
        {\bsquaredd{(\setcounter{@trace}{-2*\value{@a}}\the@trace)^2}}%
        {\bsquaredd{\setcounter{@trace}{-2*\value{@a}}\the@trace^2}}%
```

```
    \setcounter{@det}{{\value{@a}*\value{@a}+\value{@b}*\value{@b}}}
    \lambda = \frac{\the@x\pm\sqrt{\bsquared-4\times \the@det}}{2}
    \setcounter{@det}{4*\value{@b}*\value{@b}}
    = \frac{\the@x\pm\sqrt{-\the@det}}{2}
    } % ends \auxiliaryquadratic

\newcommand{\desolreal}[2]%
    {For the second-order differential equation,
    \[\dequreal{#1}{#2}\] the auxiliary function is
    \[\auxiliaryreal{#1}{#2}\]
    which factorizes to  \[\auxiliaryfactors{#1}{#2}\]
    and has solutions \[\lambda= #1, #2.\]
    The complementary function is \[\cfreal{#1}{#2}.\]
    } % end \desolreal

\newcommand{\cfcomplex}[2]{%
    \setcounter{@b}{#2}
    \ifthenelse{\value{@b}<0}%
        {\setcounter{@b}{-\value{@b}}}%
        {\relax}
    y=\ifthenelse{#1=0}%
        {\relax}%
        {\mathrm{e}^{#1 x}\left(}A\cos(\the@b x) + B\sin(\the@b x)
        \ifthenelse{#1=0}{\relax}{\right)}
    } % end \cfcomplex

\newcommand{\desolcomplex}[2]%
    {For the second-order differential equation,
    \[\dequcomplex{#1}{#2}\]
    the auxiliary function is
    \[\auxiliarycomplex{#1}{#2}\]
    which does not factorize, and so, using the quadratic formula solution,
   the solutions are given by
    \[\auxiliaryquadratic{#1}{#2};\] this gives the roots
    \[\lambda= #1\pm
        \ifthenelse{#2<0}%
            {\setcounter{@x}{-#2}\the@x}%
            {#2}
        \,\mathrm{i}.
    \]
    The complementary function is \[\cfcomplex{#1}{#2}.\]
    } % end \desolcomplex
```

The input. As an example the following input (with horizontal rules to separate out the solutions from the questions):

```
\usepackage{second_order.des.sty}
\begin{document}
{\large Solve the following differential equation:}
\[\dequreal{-3}{4}\]
\emph{The solution is} \hrulefill\par
\desolreal{-3}{2}\\\emph{End of solution} \hrulefill\par
{\large Solve the following differential equation:}
\[\dequcomplex{-3}{4}\] \emph{The solution is}  \hrulefill\par
\desolcomplex{-3}{4}\\\emph{End of solution} \hrulefill
\end{document}
```

produces the output in Figure 3.

Solve the following differential equation:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 12y = 0$$

The solution is

For the second-order differential equation,

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 12y = 0$$

the auxiliary function is

$$\lambda^2 - \lambda - 12 = 0$$

which factorizes to

$$(\lambda + 3)(\lambda - 4) = 0$$

and has solutions

$$\lambda = -3, 4.$$

The complementary function is

$$y = A\mathrm{e}^{-3x} + B\mathrm{e}^{4x}.$$

End of solution

Solve the following differential equation:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6\frac{\mathrm{d}y}{\mathrm{d}x} + 25y = 0$$

The solution is

For the second-order differential equation,

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6\frac{\mathrm{d}y}{\mathrm{d}x} + 25y = 0$$

the auxiliary function is

$$\lambda^2 + 6\lambda + 25y = 0$$

which does not factorize, and so, using the quadratic formula solution, the solutions are given by

$$\lambda = \frac{-6 \pm \sqrt{6^2 - 4 \times 25}}{2} = \frac{-6 \pm \sqrt{-64}}{2};$$

this gives the roots

$$\lambda = -3 \pm 4\,\mathrm{i}.$$

The complementary function is

$$y = \mathrm{e}^{-3x}\left(A\cos(4x) + B\sin(4x)\right).$$

End of solution

Figure 3. Example of second-order differential equations and their solutions

Cancelling fractions

Sometimes it is useful to write down the numerator and the denominator of a fraction, and for the common factors to be removed, so that the numerator and denominator are relatively prime.

In this package, the cancellation is done automatically, reducing the numerator and denominator to being relatively prime to each other. If the denominator is 1, then a whole number is printed; it also takes care of the signs.

The programming is semi-efficient; efficient in that it finds the minimum of the denominator and numerator, and finds factors up to the square root of this minimum; inefficient in that it tries every number between 2 and this minimum value, so there is some duplication (e.g. it tries 6 as a factor, after it has tried both 2 and 3).

Coding.

```
\NeedsTeXFormat{LaTeX2e}[1994/06/01]

\RequirePackage{ifthen,calc}
```

```
\newcounter{@a}\newcounter{@b}
\newcounter{@min}\newcounter{@i}
\newcounter{@c}\newcounter{@d}
\newcounter{@temp}

\newboolean{@minus}
\newboolean{@less}
\newboolean{@found}
\setboolean{@minus}{false}
\newboolean{@factor}
\setboolean{@factor}{true}
\newboolean{cancellation} % flag to tell user whether
                          % a fraction has been simplified

\newcommand{\Isfactor}[2]%
    {\setcounter{@c}{#1}\setcounter{@d}{#2}
    \setcounter{@temp}{(\value{@c}/\value{@d})*\value{@d}}
    \ifthenelse{\value{@temp}=\value{@c}}%
        {\setboolean{@factor}{true}}%
        {\setboolean{@factor}{false}}
    } %% ends \Isfactor


\newcommand{\Fraction}[2]%
    {\setcounter{@a}{#1}\setcounter{@b}{#2}
    \setboolean{cancellation}{false}
    \setboolean{@found}{true}
    \setboolean{@less}{true}
    \setcounter{@i}{2} %% to start the first factor
    \Findabsmin{\value{@a}}{\value{@b}}
    \whiledo{\boolean{@less}}%
        {\whiledo{\boolean{@found}}%
            {\Isfactor{\value{@a}}{\value{@i}}
            \ifthenelse{\boolean{@factor}}%
                {\Isfactor{\value{@b}}{\value{@i}}
                \ifthenelse{\boolean{@factor}}%
                    {\setcounter{@a}{\value{@a}/\value{@i}}
                    \setcounter{@b}{\value{@b}/\value{@i}}%
                    \setboolean{cancellation}{true}}%
                    {\setboolean{@found}{false}}
                }
                {\setboolean{@found}{false}}
            }
        \addtocounter{@i}{1}\setboolean{@found}{true}
        \setcounter{@temp}% %% check if less than square
            {\value{@i}*\value{@i}-\value{@min}-1}
        \ifthenelse{\value{@temp}<0}%
            {\relax}%
            {\setboolean{@less}{false}}
        }
    \ifthenelse{\boolean{@minus}}
               {\setcounter{@a}{-\value{@a}}}{\relax} %%  minus sign
    \ifthenelse{\value{@b}=1}{\the@a}{\frac{\the@a}{\the@b}}
    }  % ends \Fraction

\newcommand{\Findabsmin}[2]%% to find the minimum absolute value
                           %% of two numbers
    {\setcounter{@a}{#1}\setcounter{@b}{#2}
    \ifthenelse{\value{@a}<0}%
```

```
        {\setcounter{@a}{-\value{@a}}\setboolean{@minus}{true}}%
        {\setboolean{@minus}{false}}
    \ifthenelse{\value{@b}<0}%
        {\setcounter{@b}{-\value{@b}}
        \ifthenelse{\boolean{@minus}}%
            {\setboolean{@minus}{false}}
            {\setboolean{@minus}{true}}}%
        {\relax}
    \ifthenelse{\value{@a}<\value{@b}}%
        {\setcounter{@min}{\value{@a}}}%
        {\setcounter{@min}{\value{@b}}}
    }  %% ends \Findabsmin
```

Example of input and output. For this input:

```
\usepackage{fractioncancellation}
\begin{document}
\section{Textstyle}
    $\frac{16}{24}=\Fraction{16}{24},
     \quad \frac{270}{-75}=\Fraction{270}{-75},
     \quad \frac{512}{64}=\Fraction{512}{64},
     \quad \frac{-4800}{364}= \Fraction{-4800}{364}
    $
\section{Displaystyle}
    \[\frac{125}{-25}= \Fraction{125}{-25},
      \quad \frac{-120}{-36}= \Fraction{-120}{-36},
      \quad \frac{4800}{364}= \Fraction{4800}{364}
    \]
\end{document}
```

the output is shown in Figure 4.

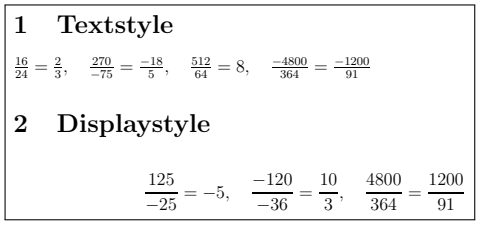

Figure 4. Example of fraction cancellation

Giving marks and comments on a student's script

I wanted to create a style sheet for giving comments and marks on a student's script. The marks for each section are known in advance, so the LaTeX package checks that the marks given do not exceed the maximum for that part, gives a comment if no marks have been given for a part, and adds up the marks for each section, and ultimately for the whole assignment.

Coding.

```
\NeedsTeXFormat{LaTeX2e}
\ProvidesClass{msxr209tma02}[2007/12/08 v1.1 (John Trapp)]
\LoadClass[fleqn,12pt]{article}
\RequirePackage{fancyhdr}
\RequirePackage{graphicx}
\RequirePackage{amsmath}
\RequirePackage[a4paper]{geometry}%
```

```
\RequirePackage{calc}
\RequirePackage[pageshow]{supertabular}
\RequirePackage{varioref}

\def\lastpage@putlabel{\addtocounter{page}{-1}%
   % borrowed from the lastpage package
   \immediate\write\@auxout{\string
   \newlabel{LastPage}{{}{\thepage}}}%
   \addtocounter{page}{1}}

\AtEndDocument{\comments%
   \clearpage\lastpage@putlabel}%


\newcommand{\@firstname}{\relax}
\newcommand{\@surname}{\relax}
\newcommand{\@studentpin}{\relax}
\newcommand{\@tutor}{\relax}
\newcommand{\@signoff}{\relax}
%%
\addtolength{\parskip}{1ex}

\setlength{\headheight}{15pt}

\newcommand{\firstnameis}[1]{\renewcommand{\@firstname}{#1}}
\newcommand{\surnameis}[1]{\renewcommand{\@surname}{#1}}
\newcommand{\studentpinis}[1]{\renewcommand{\@studentpin}{#1}}
\newcommand{\tutoris}[1]{\renewcommand{\@tutor}{#1}}
\newcommand{\signoff}[1]{\renewcommand{\@signoff}{#1}}

%% set up the header and footer

\pagestyle{fancy}
\lfoot{}\rfoot{}                 %Empty header and footer boxes
\fancyheadoffset[L,R]{\marginparsep}
\fancyhead{} \fancyfoot{}
\lhead{\@firstname\ \@surname\ \@studentpin}
\rfoot{\today}
\lfoot{\@tutor}
\rhead{\textrm{Page\ \thepage \ of\ \pageref{LastPage}}}
%\cfoot{{\sc\name\space\pin}}
\renewcommand{\footrulewidth}{0.2pt}
\setcounter{secnumdepth}{3}

%% Set up text width and height.

\geometry{textwidth=160mm}
\geometry{textheight=250mm}
\geometry{marginparwidth=00mm}
\geometry{lmargin=20mm,marginparsep=0mm}
\geometry{includehead,reversemp}%%,includemp

\raggedbottom
\setlength{\parindent}{0pt}

\renewcommand\today{\number\day\space \ifcase\month\or
  January\or February\or March\or April\or May\or June\or
  July\or August\or September\or October\or November\or December\fi
  \space \number\year} %% to give the date in UK format
```

```
% Set up default comment text,  zero marks, description label and mark total

\def\commentdefault #1#2#3{\begingroup \expandafter \endgroup%
\expandafter \commentdefault@next
  \csname #1@text\expandafter\endcsname
    \csname #1@marktotal\expandafter\endcsname
    \csname #1@mark\expandafter\endcsname
    \csname #1@error\expandafter\endcsname
    \csname #1@line\endcsname
    {#2}{#3}{#1@@mark}}

\def\commentdefault@next #1#2#3#4#5#6#7#8{%
    \def #1{No comment}% set comments for subsection from supplied parameter
    \def #2{#6}% set total mark for subsection using parameter supplied
    \def #3{0}% set default mark as 0
    \def #4{No mark}% set default comment if no mark has been entered
    \def #5{#7  &  \noexpand{#1} &  \noexpand{#3}/#2 & \noexpand{#4}\\}
    \newcounter{#8}\setcounter{#8}{0}% set default mark as 0
    }

\commentdefault{definition}{3}{Description}
\commentdefault{aspect}{2}{Aspect}
\commentdefault{outline}{5}{Outline}
\commentdefault{assumptions}{7}{Assumptions}
\commentdefault{variables}{5}{Variables}
\commentdefault{formulation}{13}{Formulation}
\commentdefault{solve}{6}{Solve the model}
\commentdefault{drawgraphs}{2}{Draw graphs}
\commentdefault{deriveresults}{2}{Derive results}
\commentdefault{collectdata}{3}{Collect data}
\commentdefault{describeinwords}{7}{Describe}
\commentdefault{decisioncompare}{3}{Compare}
\commentdefault{comparereality}{6}{Compare to reality}
\commentdefault{criticizemodel}{11}{Criticize model}
\commentdefault{describerevision}{3}{Description}
\commentdefault{revision}{7}{Formulation}
\commentdefault{conclusions}{5}{Conclusions}
\commentdefault{basicpresentation}{5}{Basic}
\commentdefault{discretionary}{5}{Discretionary}

% Set up construct for comments
\def\comment #1#2#3%
{\begingroup \expandafter \endgroup%
\expandafter \comment@next
    \csname #1@text\expandafter\endcsname
    \csname #1@marktotal\expandafter\endcsname
    \csname #1@mark\expandafter\endcsname
    \csname #1@error\endcsname
    {#2}{#1@@mark}{#3}}

    \def\comment@next #1#2#3#4#5#6#7{%
    \def #1{#5}% \wibble@text
    \def #3{#7}% \wibble@mark
    \def #4{\ifnum #7 > #2 {Error} \else \relax \fi}% \wibble@error
    \setcounter{#6}{#7}%
}

\parindent 0pt

\newcounter{@onemark}
```

```
\newcounter{@twomark}
\newcounter{@threemark}
\newcounter{@fourmark}
\newcounter{@fivemark}
\newcounter{@sixmark}
\newcounter{@sevenmark}
\newcounter{@eightmark}

\newcounter{@total}
\setcounter{@total}{100}
\renewcommand{\arraystretch}{1.5}
%\shrinkheight{-1cm}

\newcommand{\@overallcomments}{No overall comments}
\newcommand{\overallcomments}[1]{\renewcommand{\@overallcomments}{#1}}

\newcommand{\@postscript}{\relax}
\newcommand{\postscript}[1]{\renewcommand{\@postscript}{#1}}

\newcommand{\subheading}[1]{\multicolumn{3}{l}{\bf #1}& \\}
\newcommand{\subtotal}[1]{%
             &\multicolumn{2}{r}{Sub-total}&{\large \bf #1}\\ \hline}

\tabletail{\multicolumn{4}{r}{\em continued on next page}\\}
%% to show continuation at bottom of each page
\tablelasttail{&&&\\} % to avoid continuation message at end of table

\newcommand{\comments}%% The main command to assemble all the material.
{Dear \@firstname,\par \@overallcomments
\begin{center}
\@signoff\\[1in]
\end{center}
\begin{flushright}
 \@tutor\\[1cm]
\end{flushright}
\textbf{\large Comments and marks on each section}\par%

%% set up the counters for the marks at each subsection, and for the total

\setcounter{@onemark}{\value{definition@@mark}+\value{aspect@@mark}}
\setcounter{@twomark}{\value{outline@@mark}+\value{assumptions@@mark}+
 \value{variables@@mark}+\value{formulation@@mark}}
\setcounter{@threemark}{\value{solve@@mark}+\value{drawgraphs@@mark}+
 \value{deriveresults@@mark}}
\setcounter{@fourmark}
  {\value{collectdata@@mark}+\value{describeinwords@@mark}+
  \value{decisioncompare@@mark}}
\setcounter{@fivemark}
  {\value{compareresults@@mark}+\value{criticizemodel@@mark}}
\setcounter{@sixmark}
  {\value{describerevision@@mark}+\value{revision@@mark}}
\setcounter{@sevenmark}{\value{conclusions@@mark}}
\setcounter{@eightmark}
  {\value{basicpresentation@@mark}+\value{discretionary@@mark}}
\setcounter{@total}{\value{@onemark}+\value{@twomark}+
 \value{@threemark}+\value{@fourmark}+\value{@fivemark}+
 \value{@sixmark}+\value{@sevenmark}+\value{@eightmark}}


%% construct the table of comments and marks
```

```
\begin{supertabular*}{\linewidth}{lp{4.5in}rr}\hline
\subheading{Specify the purpose of the mathematical model}
    \definition@line \\
    \aspect@line\\
 \subtotal{\the@onemark}
\subheading{Create the  model}
    \outline@line \\
    \assumptions@line \\
    \variables@line \\
    \formulation@line \\
    \subtotal{\the@twomark}
\subheading{Do the mathematics}
        \solve@line \\
        \drawgraphs@line \\
        \deriveresults@line \\
    \subtotal{\the@threemark}
\subheading{Interpret the results}
        \collectdata@line \\
        \describeinwords@line \\
        \decisioncompare@line \\
    \subtotal{\the@fourmark}
\subheading{Evaluate the model}
        \comparereality@line \\
        \criticizemodel@line \\
    \subtotal{\the@fivemark}
\subheading{Revise the model}
        \describerevision@line \\
        \revision@line \\
    \subtotal{\the@sixmark}
\subheading{Conclusions}
        \conclusions@line \\
    \subtotal{\the@sevenmark}
\subheading{Presentation}
    \basicpresentation@line \\
  \discretionary@line\\
 \subtotal{\the@eightmark}
    &\multicolumn{2}{r}{\textbf{\large Total}} &
    \textbf{\large \the@total/100}\\ \cline{3-4}
  \end{supertabular*}
\@postscript}
```

The template to be populated by the tutor.

```
\documentclass{msxr209tma02}

\signoff{Best wishes,} %% Whatever closing (centred)
% statement that one wants
\tutoris{John Trapp\\00953618} %% In a flushright environment,
% so can have multiple lines

\firstnameis{} %% student first name
\surnameis{} %% student surname
\studentpinis{} %% student PIN

\begin{document}

\overallcomments{} %% overall comments before detailed comments and marks,
% closed by \signoff and tutor's name

%% Write the comments for the subsection as the second parameter,
```

```
% and the marks as the third parameter; the first parameter
% being the subheading to which it refers. The total marks for
% that section are commented out at the end, as a reminder.

\comment{definition}{}{}%{3}
\comment{aspect}{}{}%{2}
\comment{outline}{}{}%{5}
\comment{assumptions}{}{}%{7}
\comment{variables}{}{}%{5}
\comment{formulation}{}{}%{13}
\comment{solve}{}{}%{6}
\comment{drawgraphs}{}{}%{2}
\comment{deriveresults}{}{}%{2}
\comment{collectdata}{}{}%{3}
\comment{describeinwords}{}{}%{7}
\comment{decisioncompare}{}{}%{3}
\comment{comparereality}{}{}%{6}
\comment{criticizemodel}{}{}%{11}
\comment{describerevision}{}{}%{3}
\comment{revision}{}{}%{7}
\comment{conclusions}{}{}%{5}
\comment{basicpresentation}{}{}%{5}
\comment{discretionary}{}{}%{5}

\postscript{} %% To add anything after the table of comments and marks.

%% All these comments and marks are assembled outwith this file.
\end{document}
```

An example in use. To reduce the length, the comments are minimal; I have also included examples of marks not being given for a section, and too many marks in a sub-part.

With this input

```
\documentclass{msxr209tma02}

\signoff{Best wishes,} %% Whatever closing statement (centred) that one wants
\tutoris{John Trapp\\00953618} %% In a flushright environment,
                               %% so can have multiple lines

\begin{document}

\firstnameis{Andrew}
\surnameis{Aardvark}
\studentpinis{12345678}

\overallcomments{You have written an excellent modelling report,
  and you seem to have a good idea of how to model.
  I particularly liked your presentation of the graphs and figures.}

\comment{definition}{You have stated the problem very well\ldots}{2}%{3}

\comment{aspect}{You had some good ideas,
  but I thought that you should have mentioned \ldots
  }{4}%{2}

\comment{outline}{A very good outline of the problem,
  and gives me a good idea of what you will be doing}{5}%{5}

\comment{assumptions}{You seem to have covered all
  the assumptions}{6}%{7}
```

```
\comment{variables}{You had some good ideas,
  but I thought that you should have mentioned \ldots
  }{5}%{6}

\comment{formulation}{You had some good ideas,
 but I thought that you should have mentioned \ldots }{13}%{13}
\comment{solve}{Ooooh}{4}
\comment{drawgraphs}{Oh good}{2}%{2}
\comment{deriveresults}{Good picture}{2}%{2}
\comment{collectdata}{Useful data that you have collected}{2}%{3}
\comment{describeinwords}{Good description of the solution }{6}%{7}
\comment{decisioncompare}{Not really come to grips with this}{1}%{3}
\comment{comparereality}{Sort of on the right lines}{4}%{6}
\comment{criticizemodel}{Very good criticism,
  and useful comments; however, you should refer
  to your evaluation and try to assess which
  revision will help you to obtain a better model}{9}%{11}

\comment{conclusions}{Uggh}{4}
\comment{basicpresentation}{Very good presentation;
  you have been very clear in your description,
  and your use of Mathcad is exemplary}{7}%{5}

\comment{discretionary}{You had some good ideas,
  but I thought that you should have mentioned \ldots }{4}%{5}

\postscript{This is a section just in case one wants
  to add something after the detailed marks and comments}

%% All these comments and marks are assembled outwith this file.
\end{document}
```

the output is given in Figures 5, 6 and 7.

Notes

1. Note to non-mathematicians: these definitions are useful, and not arbitrarily plucked out of the ether.

John Trapp

Dear Andrew,

You have written an excellent modelling report, and you seem to have a good idea of how to model. I particularly liked your presentation of the graphs and figures.

Best wishes,

John Trapp
00953618

Comments and marks on each section

Specify the purpose of the mathematical model

Description	You have stated the problem very well...	2/3	
Aspect	You had some good ideas, but I thought that you should have mentioned ...	4/2	Error
		Sub-total	**6**

Create the model

Outline	A very good outline of the problem, and gives me a good idea of what you will be doing	5/5	
Assumptions	You seem to have covered all the assumptions	6/7	
Variables	You had some good ideas, but I thought that you should have mentioned ...	5/5	
Formulation	You had some good ideas, but I thought that you should have mentioned ...	13/13	
		Sub-total	**29**

continued on next page

Figure 5. First page of marking example

Do the mathematics			
Solve the model	Ooooh	4/6	
Draw graphs	Oh good	2/2	
Derive results	Good picture	2/2	
		Sub-total	**8**
Interpret the results			
Collect data	Useful data that you have collected	2/3	
Describe	Good description of the solution	6/7	
Compare	Not really come to grips with this	1/3	
		Sub-total	**9**
Evaluate the model			
Compare to reality	Sort of on the right lines	4/6	
Criticize model	Very good criticism, and useful comments; however, you should refer to your evaluation and try to assess which revision will help you to obtain a better model	9/11	
		Sub-total	**13**
Revise the model			
Description	No comment	0/3	No mark
Formulation	No comment	0/7	No mark

continued on next page

John Trapp 12 October 2009
00953618

Figure 6. Second page of marking example

		Sub-total	**0**
Conclusions			
Conclusions	Uggh	4/5	
		Sub-total	**4**
Presentation			
Basic	Very good presentation; you have been very clear in your description, and your use of Mathcad is exemplary	7/5	Error
Discretionary	You had some good ideas, but I thought that you should have mentioned ...	4/5	
		Sub-total	**11**
		Total	**80/100**

This is a section just in case one wants to add something after the detailed marks and comments

Figure 7. Last page of marking example