MetaPost: Numerical engines

Abstract
After years of talks about future plans for MetaPost 2.0, finally real progress is being made. This paper introduces a pre-release of MetaPost 2 that can optionally use IEEE floating point for its internal calculations instead of the traditional 32-bit integers.

Introduction

I am sure some readers are curious to know why it is taking so long before MetaPost 2 comes out, considering that I have been giving talks on the subject for years now. To get started, a recap from the initial project proposal dating back to May 2009:[1]

> In the original MetaPost library proposal we wrote in May 2007, one of the big user-side problem points that was mentioned was this:
>
> - All number handling is based on fractions of a 32-bit integer. User input often hits one of the many boundaries that are a result of that. For instance, all numbers must be smaller than 16384, and there is a noticeable lack of precision in the intersection point calculations.
>
> The current proposal aims to resolve that issue once and for all. The goal is to replace the MetaPost internal 32-bit numeric values with something more useful, and to achieve that goal the plan is to incorporate one of these libraries:
>
> | | |
> |---|---|
> | GNU MPFR library | `http://www.mpfr.org/` |
> | IBM decNumber | `http://www.alphaworks.ibm.com/tech/decnumber` |
>
> We have not decided yet which one. MPFR will likely be faster and has a larger development base, but decNumber is more interesting from a user interface point of view because decimal calculus is generally more intuitive. For both libraries the same internal steps need to be taken, so that decision can be safely postponed until a little later in the project. The final decision will be based on a discussion to be held on the MetaPost mailing list.

Since then, there has been a small change to that statement: MetaPost 2 will in fact contain four different calculation engines at the same time:

- scaled 32-bit (a.k.a. compatibility mode)
- IEEE floating point (a.k.a. double)
- MPFR (arbitrary precision, binary)
- decNumber (arbitrary precision, decimal)

The internal structure of the program will also allow further engines to be added in the future.

The traditional scaled 32-bit engine is the default, thus retaining backward compatibility with older versions of MetaPost. The other engines will be selected using a command line switch.

Working backwards from that final goal, some sub-projects could be formulated.

> - Because values in any numerical calculation library are always expressed as C pointers, it is necessary to move away from the current array-based

1. This article is an updated version of 'MetaPost 1.750: Numerical engines'. Published in TUGboat, Volume 32.2, pages 136–138. The current content reflect Metapost 1.803.

memory structure with overloaded members to a system using dynamic allocation (using malloc()) and named structure components everywhere, so that all internal MetaPost values can be expressed as C pointers internally.
As a bonus, this removes the last bits of static allocation code from MetaPost so that it will finally be able to use all of the available RAM.

This first sub-project was a major undertaking in itself, and was finally completed when MetaPost 1.5 was released in July 2010.

The current 1.80x release of MetaPost implements most of two other sub-project goals (in fact so far only the PostScript backend has been updated):

- ☐ An internal application programming interface layer will need to be added for all the internal calculation functions and the numeric parsing and serialization routines. All such functions will have to be stored in an array of function pointers, thus allowing a start-up switch between 32-bit backward-compatible calculation and the arbitrary precision library.
  As a bonus, this will make it possible to add more numerical engines in the future.
- ☐ The SVG and PostScript back-ends need to be updated to use double precision float values for exported points instead of the current 32-bit scaled integers.
  In the picture export API, doubles are considered to be the best common denominator because there is an acceptable range and precision and they are simple to manipulate in all C code. This way, the actual SVG and PostScript backend implementations and the Lua bindings can remain small and simple.

So, not accounting for hunting for bugs and fixing documentation, there is only one large step that remains to be taken before MetaPost 2 can be released, namely the actual integration of the two arbitrary precision libraries. That is why the version is set at 1.80x at the moment.

Some internal stuff

One thing that is not immediately obvious from the project goals as written above is that moving all the core arithmetic operations into functions that must be swappable instead of resolved at executable compilation time meant a whole lot of editing work, almost none of which could be automated. This is the main reason why everything took so long. Let me illustrate that with an example.

An example: a simple procedure

Let's look at the trans procedure, that applies a transform to a pair of coordinates. It calculates the following formula:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} txx & tyx \\ txy & tyy \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} tx \\ ty \end{pmatrix}$$

First, here is the original pascal implementation of that function:

```
procedure trans(p,q:pointer);
var v:scaled; {the new |x| value}
begin
  v:=take_scaled(mem[p].sc,txx)+take_scaled(mem[q].sc,txy)+tx;
  mem[q].sc:=take_scaled(mem[p].sc,tyx)+take_scaled(mem[q].sc,tyy)+ty;
  mem[p].sc:=v;
end;
```

The meaning of all those variables:

p,q	The variables for the x and y coordinates that have to be transformed
txx, txy, tyx, tyy, tx, ty	The six components of the transformation matrix, in global variables
v	An intermediate value that is needed because p cannot be updated immediately: its old value is used in the calculation of the new q
mem[]	The statically allocated memory table where Pascal MetaPost stored all its variables
mem[].sc	The structure object that holds the scaled value of a variable
take_scaled(a,b)	This function calculates $p = \lfloor (a \cdot b)/2^{16} + \frac{1}{2} \rfloor$

Table 1. Pointless caption

In the conversion of MetaPost from Pascal web to C web (in version 1.2), not all that much has changed:

```
static void mp_trans (MP mp,pointer p, pointer q) {
  scaled v; /* the new |x| value */
  v=mp_take_scaled(mp, mp->mem[p].sc,mp->txx)+
    mp_take_scaled(mp, mp->mem[q].sc,mp->txy)+mp->tx;
  mp->mem[q].sc=mp_take_scaled(mp, mp->mem[p].sc,mp->tyx)+
    mp_take_scaled(mp, mp->mem[q].sc,mp->tyy)+mp->ty;
  mp->mem[p].sc=v;
}
```

The only big difference here is the use of a global `mp` object instead of global variables. MetaPost 1.5 uses dynamic allocation instead of the `mem` array, and that makes the function a lot easier to understand:

```
static void mp_trans (MP mp, scaled * p, scaled * q) {
  scaled v;     /* the new |x| value */
  v = mp_take_scaled (mp, *p, mp->txx) +
    mp_take_scaled (mp, *q, mp->txy) + mp->tx;
  *q = mp_take_scaled (mp, *p, mp->tyx) +
    mp_take_scaled (mp, *q, mp->tyy) + mp->ty;
  *p = v;
}
```

It would be great if that could stay, but unfortunately, when numerical variables become objects, it is no longer allowed to use the simple C + operator for addition. In turn, that means that more local variables are needed to store intermediate results. To make matters even worse, these local variables have to be allocated and released.

The end result is that the same function looks like this since MetaPost 1.750:

```
static void mp_number_trans (MP mp, mp_number p, mp_number q) {
  mp_number pp, qq;
  mp_number r1, r2;
  new_number (pp);
  new_number (qq);
  new_number (r1);
  new_number (r2);
  take_scaled (r1, p, mp->txx);
  take_scaled (r2, q, mp->txy);
  number_add (r1, r2);
  set_number_from_addition(pp, r1, mp->tx);
  take_scaled (r1, p, mp->tyx);
  take_scaled (r2, q, mp->tyy);
  number_add (r1, r2);
  set_number_from_addition(qq, r1, mp->ty);
  number_clone(p,pp);
  number_clone(q,qq);
  free_number (pp);
  free_number (qq);
  free_number (r1);
  free_number (r2);
}
```

The variables r1, r1, pp and qq exist only for storing intermediate results. To be honest, qq is not really needed, but it adds a nice bit of symmetry and the overhead is neglectable.

The new arithmetic functions do not return a value since that would force the introduction of even more new_number and free_number calls. Instead, they adjust their first argument. Stripped down to only the actual actions, the function looks like this:

```
  r1 = p * mp->txx;
  r2 = q * mp->txy;
  r1 = r1 + r2;
  pp = r1 + mp->tx;

  r1 = p * mp->tyx;
  r2 = q * mp->tyy;
  r1 = r1 + r2;
  qq = r1 + mp->ty;

  p = pp;
  q = qq;
```

Where the first four lines match the first statement in the previous versions of the function, the next four lines the second statement, and the last two lines do the final assignments.

In the listing above, all those identifiers like new_number and take_scaled are not really functions. Instead, they are C preprocessor macros with definitions like this:

```
#define take_scaled(R,A,B)  (mp->math->take_scaled)(mp,R,A,B)
```

Here the right-hand side take_scaled is one of the function fields in the structure mp->math. Each of the arithmetic engines defines a few dozen such functions each for its own type of mp_number. With this new internal structure in place adding a new arithmetic engine is not much more work than defining a few dozen – mostly very simple – functions.

Using the new MetaPost

As said, there are currently only two engines: scaled 32-bit and IEEE double. Switching to IEEE double is done on the command-line by using

```
   mpost --numbersystem=double mpman
```

It is not (yet?) possible to change the numerical engine, but to make it possible to do tests inside of macros, there is a new read-only internal variable called numbersystem that returns the current engine as a string, either scaled or double.

Warning checks

In MetaPost 1, the parameter warningcheck can be set to a positive value. This will downgrade the limit on numerical ranges from 16384 to 4096, but it has the advantage that is guards against various internal cases of overflow.

With the `double` numerical engine, numerical values can range up to 1.0E+307. The warning check could be set at something like 2.5E+306, but that is actually not the most important point for a warning to take place.

Because of the way double values are stored internally in the hardware, it is possible to store a certain range of integers *exactly*. However, when an integer value gets above a threshold (it has to fit in 52bits), precision is lost. For this reason, `warningcheck` now kicks in at this limit, which is $2^{52} \approx 4.5\text{E}{+}15$.

Input format

In `double` mode, Metapost uses an extended scanner for numerical values that accepts floating point numbers in scientific notation:

```
v := 1.23E+10;
w := -4.56e-10;
```

Both `E` and `e` are allowed (Metapost itself uses `e` when it needs to report a value) and the plus sign is optional.

Note: the extension to the value scanner means that legacy Metapost input files may fail to run correctly under the new `--numbersystem=double`. The new scanning that is done for `E` or `e` followed by a digit, plus or minus sign means that at that spot, any variable with that name will be overshadowed:

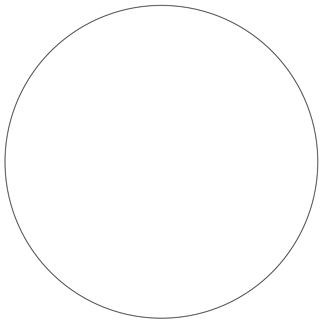
```
numeric e[];
e2 = 5;
v = 3e2;
show v;
end.
```

Normally, Metapost will show the value as `15`, but with `--numbersystem=double` it will report `300` instead. The simplest solution in cases where this happens is to add an extra `*` operator in between:

```
v = 3*e2;
```

An actual example

```
beginfig(1);
warningcheck:=0;
path p;
p = fullcircle scaled 23.45678888E-200;
p := p scaled 1E201;
draw p;
currentpicture := currentpicture scaled .5;
endfig;
end.
```



Before you try ...

- □ Some internals, like `intersectiontimes`, do not take advantage of the extra precision yet.
- □ Not all of the memory leaks have been found and dealt with.
- □ I am fairly certain that there are yet undiscovered bugs.

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