Two Questions and Answer Sessions by Donald Knuth at FI MU

Abstract

In October 2019, the Faculty of Informatics, Masaryk University, hosted Donald Knuth as a guest who led two questions and answers sessions at this occasion, dedicated to the themes of Computer Science and art. Besides some background on these lectures, you can also find their transcripts in this article.

Keywords

Donald Knuth, Computer Programming as Art, Boundless Interests, Q&A

In October 2019, the Faculty of Informatics, Masaryk University in Brno, Czechia, celebrated the 25th anniversary of its foundation. Several events were held for this occasion, the most prestigious of which was perhaps the Week with Turing Prize Laureates (Sojka, 2019a) with Donald Knuth and Dana Stewart Scott as the guests of honor. At the end of the week, the Czech premiere of Knuth's musical opus, the Fantasia Apocalyptica oratorio, (Knuth, 2020b) took place, as discussed in the previous issue of the Cs-Tug Bulletin. (Lupták, 2019)

This was not the first time Donald Knuth has ever visited Brno: In 1996, Donald Knuth gave lectures and has received an honorary doctoral degree at the Faculty. (Zlatuška, 1996) Twenty-five years later, Donald Knuth gave two lectures in the Q&A format inspired by the famous physicist, Richard Feynman. The titles of his lectures were *Computer Programming as Art*, and *Boundless Interests*. Both lectures were moderated, the first by prof. RNDr. Jozef Gruska, DrSc., and the second by prof. RNDr. Jiří Zlatuška, CSc. Questions to the speaker were also posed by the audience via the Slido web service.

In the first lecture, Donald Knuth confided to the audience about the importance of batch processing, the advantages of not reading e-mails, the deviousness of the "What is you favorite X?" type of questions, Knuth's cooperation with the NSA, the renaissance of magnetic type data storage algorithms, and the relation between the sounds of a water fountain and a crying infant.

The second lecture was interleaved by passages from Fantasia Apocalyptica, played by Donald Knuth on a digital keyboard. In the lecture, you will learn about the musical side of Donald Knuth, the motifs used in Fantasia Apocalyptica, the reducibility of its composition to Constraint Logic Programming (CLP), how Donald Knuth grappled with burn-out during the composition, how he managed his time, the difference between programmers and *good programmers*, what Donald Knuth considers the worst program he ever witnessed, and also Donald Knuth's very own pipe organ.

Additional information, video recordings (Sojka, 2019b; Sojka, 2019c), and transcripts of the lectures (Szaniszlo, 2019a; Szaniszlo, 2019b) are available on the web of the Faculty. A video recording of the Czech premiere of Fantasia Apocalyptica is available on YouTube. (Knuth, 2020a)

Questions and Answers Session 1: Computer Programming as Art

Gruska: Good afternoon. Dear informaticians, we have today very special colloquium. We have the legend of informatics, father of analysis of algorithm, father of art of programming, father of many other books, father of TFX and so on and so on... I better stop, because I would spend probably all my time here talking about his outcome. He came here to answer your questions. Sessions will be so interesting, how interesting will be the questions you ask. There are two ways to ask questions. Either by microphone or you can use mobile. Ok?

I mentioned that this legend of informatics and one argument supporting that is when in 1989 there was the IT World Computer Congress in US in San Francisco, professor Knuth was invited to give the first talk, the most important talk. So welcome, professor Knuth. Welcome here, and you can start the session, colloquium, by asking the questions from your audience. Welcome in Brno. applause

Knuth: Prof. Gruska, do you have a question? **Gruska:** Yes! 42 years ago I wrote you letter.

Knuth: Oh!

Gruska: I was waiting for answer. After one month I decided to call you. Secretary took the phone and said: "Oh, he's three months behind answering letters. However, he is next to me." So I could talk to you. Has that changed with your custom to respond to letters?

Knuth: Can you hear me? Testing... Testing, testing. Zero one, zero one. laughter So the secretary I had 42 years ago is Phyllis Winkler who served me for many years, but she died about 15 years ago and actually retired before that. However, I go into campus at Stanford four days a week, and I have lunch with the students every Thursday and with the faculty every Tuesday and things like that... However, I found that one of the great secrets of Computer Science is called batch processing. So when I answer mail, I don't do it by interruption, but by batch processing. So I gave up email, I guess, on January 1, 1990, and I've been a happy man ever since. But the questions come in, and they're filtered, and I answer a lot of them all at once. So it turns out that that's necessary for me in order to work efficiently. But Phyllis was a wonderful protector. So she kept it possible for me to work efficiently for me all these years.

Gruska: And there's also another question, well, they ask, one student: Can you talk to Knuth? He said: "Yes, Friday night."

Knuth: Friday night? Ok.

Before I go on to more questions I want to mention that the whole inspiration for this kind of a session came about when I was at Caltech during these 1960s and Richard Feynman, our famous physics professor, would end every one of his classes the last day of class: Anybody can ask any question they wanted to. And so I started using that in my own classes, and I kept that up until I retired. And so now I find this as a best way to customise the lecture. So I will try to give an answer to basically any question, and I'll try to keep it short, so we can move to a variety of question.

On the other hand, tomorrow I'm giving another talk at 12:30, and it's also called Questions and Answers. Tomorrow I'm gonna have a piano keyboard, so that I can answer questions if they have anything to do with music because I think when you walked in here, you've got an ad for a concert that's gonna be given on Friday. So today let's not have questions about music, but anything else is fine.

Then I also brought this with me. This is a prop for a paperback book that I published, I don't know, 2-3 years ago, which is the middle third of Volume 4B of The Art of Computer Programming. So far I've finished Volume 1, 2, 3 and 4A and I'm working on Volume 4B. I started writing it in the middle, and this came out. It's called satisfiability, and the big six on it here says Volume 6, Fascicle 6. Fascicles 0, 1, 2, 3 and 4 were part of Volume 4A.

Now the reason I'm saying this is that right now, maybe as we speak, Fascicle 5 is being printed and it will be available in bookstores a month from now, and you will love it. It's a wonderful book. Ask your parents to get it for you for Christmas. *laughter* And the main thing is that it's, I would say, maybe eighty percent of it is about puzzles or topics that are often considered not only worthwhile but fun. I sort of have been waiting all my life to present this material in showing how puzzles are very relevant to learning how to be a good computer programmer. And so that's Fascicle 5 gonna be coming out in a month. So that's end of advertisement.

Gruska: There is a question: What is your favorite unsolved problem in Computer Science?

Knuth: My favorite unsolved problem in Computer Science... Ok, well... I guess, personally, what is the worst kind of question to ask? And I'm sorry, but it's a question that starts by "what is your favorite X?". Because it's really hard to say what is my favorite X: almost always "what's my favorite algorithm?", "what's my favorite... relative? – Do I like my son better than my daughter?", and so on... So I hope not too many of the questions today are gonna be "what's my favourite X?", however, I don't wanna duck this one.

So unsolved problems in Computer Science. It depends on who and whether I think it's gonna be solved or not. So in order to be a really favorite problem, it would one where I expect that when I tell you what it is, then next week someone in the room will solve it. So often my favorite problem is one that I've just thought up. And last week I thought up a problem that I mentioned to one of Dan Král's students on Sunday night, and so I'm not gonna repeat that one now, so he can work on it and solve it for me.

Of course, the most famous unsolved problem in Computer Science in a sense that it's got a million-dollar price associated with it is a question "does P equal NP or not?" Someone is gonna ask me about that anyway, so I might as well spend one or two minutes on that. So the question is: The P is the set of all problems for which we can solve in polynomial time, and NP is a set of all problems for which we can verify a solution in polynomial time, more or less. And so the big question people say is "Does P equal NP or not?" Now, I wanna amplify that question. So let's try to be specific. It's known that the question "P equals NP?" is equivalent to saying "is there a polynomial-time algorithm to solve one particular problem?" Let me take satisfiability, for example because I held that up.

So there's a problem called 3-SAT which says: I have n clauses and each clause is of the form $(a \lor b \lor c) \land (d \lor e \lor f)$ and so on. Various clauses like these and all together [logical] and of these. And a,b,c,d,e and f are actually $x \lor \bar{y} \lor z$ or something like that... a,b and all of these things are Boolean variables that are either negated or not. And the question is: Can we satisfy all these clauses simultaneously? Is there a way to say that x is true, saying y is true and z is false, and all of the clauses are gonna be true? And that it's satisfiable if and only if there is a way to set these variables.

So when you look at this problem, you say: "Oh, sure, I can easily solve this problem." However, I don't wanna say if you've solved it tell me how you did it because a lot of people have told me that, but it was a waste of time. A lot of people think they've solved this problem, but they didn't.

The question "P equals NP?" says "is there an algorithm that solves 3-SAT to some constant steps?" I wanna ask another question, and that is: Could we know an algorithm that solves 3-SAT? Some people think that these are the same questions. If an algorithm exists, certainly, we would know what it is, right? However, that is a giant step to go from saying that there is an algorithm, and there is an algorithm we actually can find. A lot of things are proved non-constructively, so it can go several ways. One is that we have "yes, there is an algorithm, yes, we know it". Or it might be "yes, there is an algorithm, but no, it actually is beyond our grasp". You can never

write it down. It exists up there, but you need to be God in order to know, actually to use it. And then there is a case "well, there is no algorithm". Well, then this [the fourth cell in a schematic combinatorial table] had better be no. I'm not gonna have an N and Y in here.

Gruska: Would you be happy to have non-constructive solution?

Knuth: Well, that's what I believe is probably true. I'm not sure how happy I would be, but it could be that the total number of algorithms is huge, and if it doesn't exist, it's a completely different question. It's quite likely, but hardly anyone thinks about this puzzle. It's quite likely that the algorithm exists but only because there's only finitely many reasons why it doesn't exists. And that's not gonna tell us much. There are many cases of non-constructive things where we know, for example, that there's a winning strategy in a game of hex, but nobody has any way to actually win hex. It's just known that there is a winning strategy for the first ...

Gruska: So next question... Knuth: Yeah... laughter

Gruska: Do you still write any code? If so, why and which programming language?

Knuth: In this case, I could even say what my favourite programming language is. laughter When I wrote Fascicle 5 I probably wrote about 600, I don't know, many hundreds of programs and every week I write on average at least five. Most of programs are rather short, of course, but some of them get to be fairly good size. The language I use all the time is called CWEB. It just really works for me. It comes installed with Linux, and so I've got many dozens of CWEB programs as examples on my website. If anybody wants more information about any of the ones that I didn't put on the website yet, I'm glad to put it online.

It's a combination of TFX and C. It's so much better than any other way to write programs, but I'd better not get started on it. To me, it's the greatest outcome of all of my work on TFX. The fact that I now have CWEB in order to solve lots of problems.

Gruska: Ok, a related question. Did you try to write program for quantum computer?

Knuth: Ahaaa! Quantum computing is something that I have absolutely no intuition for. If quantum computing turns out to be the best way to go and everybody switches over to quantum computing, in a way, that will be the best thing for me, because then I'll have more time to write my books. Because I'm never gonna write anything about quantum computing. I only promise to write about things that were known in 1962 when I started a project [The Art of Computer Programming]...

Gruska: Another question is such that you may have a difficulty to answer. The question is: What were the research problems you worked on during your cooperation with NSA? laughter

Knuth: So I spent a year before going to Stanford working on code breaking, and I'm not allowed to tell my wife what I did during that year. laughter

Gruska: She's not here! *LAUGHTER*

Knuth: But this is being recorded, and maybe she would watch the recording.

Gruska: [Disappointedly] Oh...

My question is: What was your subject in your thesis? PhD. thesis. And did your advisor help you?

Knuth: I was lucky to work with Marshall Hall at Caltech, and so my thesis was about a general area that now would be called combinatorial designs. More specifically, finite projective plane. This is an arrangement of points and lines that are abstract. There is a parameter n, and I think it was... $n^2 + n + 1$ point, and every line contains n+1 points and every point is on n+1 lines and any two lines intersect at exactly one point. So this is like a projective geometry, but finite projective geometry, instead of the projective geometry of the sphere or something. So my thesis was to show that

certain kinds of finite projective planes do exist and they hadn't been known before to exist. The whole area of designs is to find families of sets that have interesting properties that might be useful in application.

Now the interesting thing is that over the year since I graduated, I applied almost every branch of mathematics that I've ever heard of to computer programming except the theory of design. Because subsequently, we've found out that we can do better with random choices in almost all cases instead of trying to find these very rare patterns that sometimes exist. It was very good training, and my advisor helped me in the following way: I was working on another problem and one morning as I rode up in the elevator, got to my office and said hey, I betcha I can solve this particular problem I just heard about. And my advisor said: Ok, that's your thesis.

Gruska: Another question: Vim or Emacs? [The question wasn't answered at this moment. But see a later mention near the text "Tabs or spaces" in this transcript.]

Knuth: What do I think about Artificial Intelligence? Ok. Do you think it could be dangerous?

Certainly, I prefer real intelligence to Artificial Intelligence. I have always felt that the working ??? AI has been at the cutting edge of Computer Science. Of the last sixty years, the people working on the challenging problems of Artificial Intelligence have come up with many of the most important innovations in the field. I always regarded it as something that tells us how to stretch what we know and invent better algorithms rather than as something where I would actually use the algorithms afterwards and believe that

In other words, suppose we develop a really good algorithm that decides whether or not a student ought to graduate. Should I let the computer decide which students graduate or should I try to understand their working and see if it has some value?

Right now, the question about the danger is extremely pressing and especially with respect to military applications. There is this really frightening movie, short video... I can't remember the name of it. Came out about three years ago. Where you could pretty much, with today's technology, you could program drones to kill anybody you wanted to. The scenario I should remember, some of you will maybe remember the title, but my friend Stu Russell at Berkeley was one of the people behind that film. Essentially it's already possible to do these horrible things, so we gotta find some way to keep that from happening. Stu has the best idea so far about how to prepare for such dangers, but still, I'm afraid his ideas are not satisfying me because they depend on assumption that human beings are rational. And the more I read about these days, the less and less I believe that human beings are rational.

It's very serious to see how to restrain the things that we don't understand and figure out how, because with the new techniques we are able to solve many problems, wonderful problems in all branches of science that we weren't able to solve before. The ones that are encouraging to me are the ones where there's no enemy involved. Like somebody trying to defeat us in the experiment, but the only enemy is that we're battling ignorance, we're trying to find some pattern in the way stars work or something like this, the way biology works, how to identify diseases of different kinds... And these machine learning techniques are wonderful. Even though we don't understand anything about how they work. But you use the same algorithm for something where there's adversarial interest involved, then everything gets bad.

Gruska: Another question was very nice, and I think you will like it: Could you tell us the story of T_FX from the very beginning to implementation?

Knuth: Ah, yes. *laughter* Yes. In short, I found out that computer technology had changed so that the work that was once done by hand with hot metal was no longer being done. It was replaced by new technology which was based on photography. And the new technology looked awful. I saw the proofs for the new edition of Volume 2 of The Art of Computer Programming, and I was almost sick. I didn't wanna have a book that looked like that. And then a couple of weeks later I learned that computers

might be the answer, because somebody working in Southern California had found out that actually with digital methods, with pixel zeros and ones you could potentially make books that would look just as good as the real printed one.

So I got in an airplane and flew down to South California, talked to the people there and decided to change my sabbatical plans for the next year where I was gonna study combinatorial algorithms. Instead, I decided that oh, I can now solve the problem with the books if I only write the computer program that makes patterns of zeros and ones that say what should be on every page of the book. Well it took a little longer than a year, but that was the beginning of my work on typography.

Zlatuška: So the proofs for Volume 2. That means Volume 1 was already printed not using T_EX? Does it exist?

Knuth: No. Volume 1, 2nd edition have already come out.

Zlatuška: The first edition you ??? about.

Knuth: Volume 1 would have come out looking bad at some point, but I was revising Volume 2, and so those proofs came in. What happened, is the technology went away from hot metal, basically monotype setting, and actually in Eastern Europe was the only place left for people who were still doing monotype during the 70s. So I have a translation of my book in Hungarian that as done by these hand methods still look good in the 70s, but the books that were printed... if you look at all the journals of mathematics that were printed in the 70s you see what I mean. So I was feeling bad about until I realised it was just a matter of programming. The machines were there that would make the book, but I need to get the pixels figured out. So I spent a long time in the library looking at everything that I could see about how to make good-looking books, and I brought the experts to Stanford, and we worked together on it for a while.

Gruska: How many months you worked on T_FX?

Knuth: It's hard to say because I was also doing a few other things, but at one point I took a leave of absence from Stanford for a year because I found that working on software was harder than writing books. You can write books, you can write papers, but software involves much more of your brain, and I couldn't swap in and swap out so much without taking a year off of teaching and getting TFX done. Then I could go back and resume the other schedule. But in calendar time I finished this five-volume set of books called Computers in Typography. I finished that in 1984, I have started the project in 1977. So that's seven years. Then I came back to it in 1989. I came back to it because I didn't realise that people were gonna be using it for typesetting strange languages like Czech. *laughter* You know, you have accents on letters. Wow. So I went from 7-bit to 8-bits in 1989. Took a year.

Gruska: As far as I remember, when working on TFX every evening or every morning you wrote down what was good, what was bad. Did you make use of these comments?

Knuth: Well, I got a paper out of it. I have a paper called "The Errors of TEX" which I think is a good idea for everybody - to keep track of what mistakes you make. Programming is too complicated, you can't get it right all. And I wanted to find out what were the kind of errors that I made and also learn something about patterns and that I could change my actions. I kept this log showing every century(???), every major non-trivial change that I made, some of the trivial ones too, to TFX and to METAFONT over the years. Then I was able to get a good understanding of the scale, how important are certain kind of errors. For example, people say: goto statements are bad. Look at my history with T_FX – sure enough, I made some errors, because I used goto statements improperly. However, I had also used every other kind of statement improperly too. Every statement - assignment statements are bad, conditional statements are bad, everything from that aspect can be misused. But I did learn something about which kinds of things to avoid and the corrections were not only to fix errors but also to improve user interface and things like that.

Gruska: Next two questions are pretty ??? so please ???.

Knuth: All right so... What would you advise to your 25 years [old your]self?

So 25 year, that would be... 1963. That's the year I got my PhD. ... I decided that year to become a college professor. I actually been offered the year before when I was 24 to drop out of grad school. Essentially I was offered a salary of \$100,000 a year plus an assistant. Now in 1962 that would be like \$10,000,000. It's not anywhere near Bill Gates's salary, but anyway I knew that my role in life, my interest was going to be to work with students rather than to maximize the amount of income that I had. On the other hand, I'm not saying everybody should make that decision. Anyway, at 25, that's when you want to start becoming stable and making long-term decisions that you're gonna live with.

Knuth: Next question: Anonymous:

Gruska: How being a Christian affected you as computer scientist?

Knuth: It's hard for me to say what it would have been like if I had been born into a different family, but certainly, I guess, one of the ways, my Christian upbringing with respect to work on crypto and questions of computer security. I'm not very good at doing work on cryptography, because I'm not as sneaky as the people who are trying to defeat these things, but with respect to security all my life I had this idea... I'm sorry, not security, privacy... I always had the idea that everything that I do is known to God, so I don't have absolute privacy. And I didn't understand until later that there are people who think that nobody should know what their thoughts are. So it makes it harder for me to understand, but some people concern about privacy, although of course, I don't want my thoughts to be used by the devil, by something that's going to exploit me, but I'm not very comfortable if there was something beneficent watching over what I'm doing.

All these things, I guess I should say, I'm very happy that there are parts of my life in which there's no mystery, and I can prove that something is right, but I wouldn't be happy if there was no mystery whatsoever, so I appreciate the fact that there are things that I'll never understand, so it teaches me some humility that I shouldn't expect to understand everything. So I don't claim to understand everything, and I'm very happy that God did not make it possible to prove or disprove the existence of God.

Gruska: Related to this question: Let us assume that you will live still 30 years.

Knuth: Do I? What now? ... Oh goodness. *laughter*

Gruska: By famous visionary Kurzweil at that time we should have laptop with information processing power better than all human brains. What would you do with such laptop?

Knuth: I would certainly try to finish The Art of Computer Programming. Let me rephrase your question. How do I want to continue, you know, what should I do the next week and the week after that? How do I want to continue to live? I have to be watching when am I gonna start going senile. At the moment I don't think I've reached that point yet, but it might come to the point where I should stop writing The Art of Computer Programming because I'm starting to write stupid stuff.

Gruska: In which area of Computer Science or mathematics do you see the most potential?

Knuth: In which area... This is one of these favourite questions again. The much harder question would be: "Which area does not have much potential?" because I think everywhere I look, I see potential. The problem is really have to go to sleep at night not using the knowledge we have. Everywhere I look, I see that it's not saturated yet.

Gruska: Additional question is pretty philosophic: What your opinion on Curry-Lambek correspondence? Do you think mathematics is constructed or exists independently?

Knuth: I guess I'm a Platonist in the sense that I'm discovering things that are there already. The stuff that's there is all consistent with my own attitude that these truths are there and I just am learning a few of them at a time. There might be an algorithm that solves 3-SAT, and that would be there. In fact, I'm not sure I even understand how I could be a non-Platonist.

Gruska: What was the most valuable ??? you learned during your career?

Knuth: Not to use email? *laughter* There we go.

Gruska: Is too late for 20 years old students to start learning real mathematics and programming?

Knuth: A 20-year old student? My goodness, no. I didn't now that much math when I was 20. It depends on what you've seen so far and the teachers you've had. But I consider life to be a binary search where you find out things that are relatively easy for you, because of the unique experiences that you've had, and you try some things, and some things work, and some things don't work. Then you keep learning more about yourself. I'm 81 years old now, I'm still not sure exactly where to go, but I keep trying different thing. I have given up on quantum computing, though. laughter. I tried to understand quantum computing, and I know people who do understand quantum computing, but I ??? it's not me.

Gruska: Do you believe in non-locality in physics?

Knuth: I understand a few things about multiverse and things like that. For example, there's no way to distinguish between whether or not this lecture I'm giving now is simultaneously forking into many different things and so each different incarnation of it will have a different series of questions, and I'll give different answers to different questions and so on. And all of this idea that there are these all the different universes all simultaneously existing is consistent with quantum mechanics.

Gruska: Biggest challenge of becoming a good programmer?

Knuth: What does it say? Spaces... Tabs or spaces? *hehehe laughter*

So I use Emacs for my hacking, and I always untabify ??? but I also read a lot of programs that other people have written, and I start out with changing all the tabs to spaces. Of course, I'm using CWEB, not Python.

Knuth: Next... Some of the exercises are known to be open research problems. Yes indeed. If the number is 46 or higher, it's something where, as far as I know, it hasn't been solved yet.

Knuth: Has anyone ever contacted you that they have solved one of them while reading the book? Yeah. People look at them ??? I do want to point out that a lot of people haven't been looking at those lately, so ... I'm sorry...

Gruska: Excuse me... We make break for 5 minutes because they need to change [the microphone].

Knuth: So we had a celebration at Berkeley, I guess a month ago, celebrating the life of Dick Karp and at that time there were a dozen speakers, and I decided I would say something about the Karp's work that the others weren't going to talk about. So I looked again at Volume 3 of The Art of Computer Programming, where Dick had told me about some things he never published that applied to sorting on tapes. Nowadays people don't use tapes for sorting, but when Volume 3 came out, this was one of the biggest topics in all of Computer Science. People were saying one third of all computer time was spent on sorting, and people had these tapes, and these were must have to have in the book. But since we don't do sorting that way anymore, I should rip out all this, I don't know, sixty pages or something of Volume 3. And then people say "no, no, don't take it away, because we're just finding out there's a new memory kind of being invented which is rather similar to tape and so these old techniques are gonna be good!"

Anyway one of the things about magnetic tape is that you can read it forward and backwards, so you can write information on the tape, and then you can read it in the other direction, and that would be much faster than rewinding and reading back forward. And Dick Karp worked out a beautiful theory about patterns of using tapes for sorting that he showed me at lunch one day and I put it in my book, and he never published it. So I showed it to the people at Berkeley, and as I was looking through this I came across a research problem, you know the level 46, which it seems to me is ripe for solution now. 50 years have gone by, and people know a lot more math than then, so I suspect that there are dozen problems in there that are just waiting to be solved. So you can go through, you have to spend a little time paging through and checking out the numbers. And if it's 46 or more then think about it and say "hm, I wonder if I can solve this now", because people haven't been doing that systematically.

Zlatuška: How many girls you seduced because of Computer Science?

Knuth: How many girls have I had thanks to Computer Science?

Gruska: Forget it. Forget it.

Knuth: So, in fact, I had 28 grad students, but none of them were female, *laughter* but I did serve on many committees and many others and so on...

Gruska: Here is better question: What's the biggest motivation that kept you going through career?

Knuth: I guess it's the example of my parents which was always to somehow be a servant, to see how I could be of use to somebody else. My father's name was Erwin. He had an informal, I guess you'd call it a startup ???, he is a one-person operation, and he would do services for all churches and schools and a few other nonprofit things like this, and he called it ERW service. He thought it was clever he could write the word service but make ERW large, so would say ERW service. But anyway, that epitomized all the philosophy that I grew up with – to be a service. If I got to a point where I thought that I couldn't be of use to anybody anymore, I told my children not to keep me alive just to make me happy, but if I get to a point where I don't recognize them or anything like this... How do I express this... There's a novel, came out less than 20 years ago by P. D. James and it was about... the world... the people discover that from now on all women in the whole world would be infertile and so there will be no more children ever born again. So humanity was eventually going to die out. So what did people do? That was very devastating to me, to imagine what it would be like if I was in a situation where I could not do something that would be of use to people, more than my immediate family, but to people in the future.

Well there's a short story by... Oh, goodness... Okay, who's the greatest Argentinan author?

Zlatuška: Borges.

Knuth: Borges, yes, of course, Borges. So he has this short story. I think it even takes place maybe in Czechoslovakia, I don't know. But anyway, it's a story of a person who is a playwright, but he's facing a firing squad, because of his political views. And just as the guns are aimed at him and so on, he prays to God, and he says: "God, I have to finish this play I'm working on. Please, make time stand still so that I can work out all the details and figure out exactly what should happen in this play." So God says okay and time stands still, and this playwright solves the problem, he figures out exactly what's the perfect play. And then he's shot. So nobody ever gets to see what happened in the play. It was just that the playwright himself was able to solve that particular problem. So that's the opposite of my own way to do it. It has to be something that I do that somebody can use. I hope that's making myself a little clearer what this idea of service is.

Gruska: There are quite a few people that try to put the idea of formal method into programming, software development... What do you think about that?

Knuth: When you're putting ideas into formal methods, it forces you to understand what the ideas are. You don't realize what you don't know until you try to formalize it some way. So it's a great educational experience. On the other hand, I guess I said a similar thing about Artificial Intelligence a while ago that I consider it as a very useful way, a source of good problems and continuing to learn. But then I was less interested in actually using the programs afterwards because I knew that they would only be approximately right.

I once had a language called SOL, Simulation Oriented Language, and the idea was that it would be it would be pretty easy to write models for discrete systems. And you could simulate the system. As I was working on this language, I looked at a lot of different applications, and in each case, I found out that the idea of formalizing the application and putting it into this language was a wonderful educational experience. I can build models of things to simulate, but I learned much more writing the model than I did actually running the model afterwards. So I almost thought I should take output statements out of the language. The people only use language for formalizing their model instead of for actually running it and believing the answers that they get

Nobody seems to understand what I'm saying, but anyway, I believe the main advantage of formalism is educational rather than actually having a payoff afterwards.

Gruska: Next question: For how long can you program or read papers per day until you are exhausted? How do you relax? Do you relax sometimes? laughter

Knuth: Yes, but in fact, I'm relaxing right now. *laughter* At night, I have to take my mind off whatever I'm doing. So I read James Bond or something not really heavy literature. I'm reading right now a novel by Frederick Forsyth called The Odessa File which is story about the German SS in Latvia... what you'd call a thriller. Some kind of story like that and I go right to sleep. I read novels at the same speed as I read a math paper. laughter I don't know any other way to do it, so I don't get through that many books. However, when I do come across a book, that I think is especially nice, I put it on my website. So I think if you look on my website and under... I think, there's a Frequently Asked Questions and it says "so you're retired" or something like this. You click on that, and it'll tell you, I think, maybe three dozen books that I think were special to me.

Gruska: Did you work hard when you were student?

Knuth: I was a machine. I was a problem-solving machine. Somebody said: Okay, Don, do this, and I would do it. And I didn't start reading for pleasure probably until I was about thirty years old. I was given an assignment, and I was a good boy. So I did it.

At least that's the way I remember how it was. I don't know how it really was. I think I also was a... they might call me a wise guy. I mean, I was cracking jokes and not paying too much attention to what I was supposed to do.

Gruska: Did you watch westerns or detective stories?

Knuth: What about detective stories?

Gruska: Did you watch westerns movie or detective stories?

Knuth: I certainly watch a lot of movies, but, by the way, I might as well mention one thing. I guess it was a month ago when I saw again a movie. It's called Double Indemnity. It came out in 1944 or something like that. It's one of the earliest noir movies, and it stars Fred MacMurray and Edward G. Robinson and Barbara Stanwyck. That movie has special significance for me because when I was writing The Art of Computer Programming, that movie was playing almost every night on The Late Show. As I was typing The Art of Computer Programming, I would have the TV on, and I would see Double Indemnity over and over again. So The Art of Computer Programming was written largely to this movie Double Indemnity. The music to Double Indemnity was written by Miklós Rózsa, and it's haunting music that I hadn't

remembered how haunting it was until about ten years ago. I saw Double Indemnity again at a theater and immediately when they showed the title, and I heard this music, and I said: "Oh my god, what powerful music is it." So I've included music from Double Indemnity in my piece that's gonna be played on Friday. Although I'm not sure if I'm gonna be sued for this. But one of the things that occurs, you can check it out by watching the movie.

Gruska: Didn't you have idea to write science fiction?

Knuth: I talked about writing it or? I enjoy different kinds of science fiction, of course, but I haven't had time to... I wrote this little book called Surreal Numbers. I kind of think of it as a little bit like opera in the sense that opera is good music to a little bit of a plot. My book Surreal Numbers is good mathematics to a little bit of a plot. The characters in this story work on a math problem together, and it makes a little plot, but mainly there's beautiful mathematics that they're discussing.

If I live long enough and finish The Art of Computer Programming, I'd like to write some science fiction. One book I'd like to write is where the story is told by ant colony. Not by an ant, but by ant colony. There are tens of thousands of ants, and somehow they cooperate with each other and so that they form a consciousness and so I think there ought to be a short novel that's told by an ant colony.

The other thing is a short story something like... you could call it The Fly. Now, Mark Twain in one of his books, I think it's called The Mysterious Stranger, there's a character in there representing the devil, and early on in the book the devil opens a window, and a fly goes out the window. And he said: "Because I let this fly out the window, there's gonna be war next year." He's predicting what they call the butterfly effect of chaos theory. All kind of things are codependent.

So maybe people have seen this movie Run Lola Run that came out of Germany a few years ago. It tells a story, three or four times the story starts out exactly the same way, but then there are three or four different, completely different endings, just because of little changes in time. So I think it would be fun to write stories that... Well, it's not one story, but it's two or three stories that all start out the same, but end completely differently. But on the other hand, did this movie, its German name is Lola rennt. Run Lola Run in English. So the author of that movie already did it, what I was planning some time to do.

Gruska: So, please, ask questions. Yes, will be finishing. Maybe those who don't have mobile with, they should have the chance also to ask questions.

Knuth: Yes!

Student: You have a trouble ??? distractions or maintaining focus when you're working on a problem during the day? And if you do, how do you sit down and focus and do some piece of work during the day? You have like a ???.

Knuth: If I understand, you're saying how can I focus on one thing instead of many others? I don't have all the distractions of modern day. I don't have the radio play, music blaring ???. I found that, for example, if my job for the day is to do something like proofreading, then it helps me to have some kind of music like Telemann or something playing in the background. It sharpens my mind. But if I have Bach playing the background, it's too much, and everything goes away. So part of it is having no distractions, no interruption.

When my children were babies, they would be crying. I had this problem: How am I gonna concentrate on writing a book when the baby is screaming? The answer was white noise. I had a waterfall, and I could turn the fountain on and crying plus white noise equal white noise. *laughter*

But in general to concentrate on one thing I collect a lot of material, all on the same subject. I'll read thirty papers about that subject all at once, instead of reading about many different things. So I spent a lot of my time filing things away to be read later. Batch processing is very important.

Sochor: My question is: Do you prefer screen and keyboard or paper and pencil?

Knuth: I love that question because it turns out that I learned when I was in college that I could write a letter home to my parents faster with pencil and paper than on a typewriter. The reason was, actually, because I'm a good typist, I type faster than I think. laughter I had actually gone to typing school, so I can type so fast that it's a synchronization problem. laughter I'm not ready for it. But pencil and paper for me is a perfect sync with my thought process. So I make the first draft of everything in pencil paper, but then I go to the computer, and I polish it, and I edit for style. And I'm typing on it ??? I do that.

Gruska: So, you don't have to think, you just looked on your fingers.

Knuth: That doesn't work for me.

Student: So my question is: How has your stay in Czechia been so far?

Knuth: In fact, I wanted to say at the very beginning that it's a thrill for me to be invited here to celebrate your 25th anniversary. It couldn't have been better in every way, and I'm really enjoying this week. I was babbling over with enthusiasm for that, but I forgot about it at the beginning.

Of course, I've only had five/six days so far, but each one has been a joy.

Gruska: Ok, I think it's time to make break. The break will be quite long. Continuation will be tomorrow. Thank you, professor Knuth. applause

Sojka: Come tomorrow and those who want to make a photo with Don and the faculty, the photography ...

Questions and Answers Session 2: Boundless Interests

organ music playing

Zlatuška: Ok, ladies and gentlemen, dear colleagues, I would like to welcome you at the second session of Questions and Answers with professor Donald Knuth. Today, again, there is a possibility to put questions over your phones or whatever connection this works (???). And today's topics are not limited to just Computer Science, but also to music.

We invited Don for 25th anniversary of Faculty of Informatics. Not only as the first honorary doctor of informatics of this university and this faculty, but also as personality who transcends boundaries, and especially we felt that it would be interesting to have him here also as a musician.

I believe he's got at home not only his work office devoted to The Art of Computer Programming and informatics, but also to music. He built his own organ in his house at Stanford, so this other Donald Knuth, I believe, will be for many people as interesting as Donald Knuth, the author of The Art of Computer Programming. If you look at that from other perspective, I feel that the idea of programming as an art—Don quoted that yesterday—is something which resonates with this second personality, but I hope that there will be also questions concerning this part.

Now. I thought Donald travelled with this, but he travelled so light, but it would be impossible so... The floor is yours with the questions. Any questions from the floor?

Knuth: Right, so, hello, everybody. I want to thank again for the wonderful hospitality this week, and before I forget, I also want to mention that there's another lecture today by Dana Scott. I think 4:30 in the afternoon. So, you know, if I give a bad lecture, you at least get one good one today. Zlatuška's laughter

The other thing I wanted... Before we start with new questions, I had a couple things to say. First of all, I don't know if you've ever given a lecture, but the night after it you wake up in the middle of night saying "Oh, I should have said that!" and so I thought of at least two things that I wish I had said yesterday.

So in the first place, I was trying to explain why the question about P versus NP is not as simple as people usually think when they talk about whether there exists an algorithm that runs in polynomial time, and they think it's the same as saying that we could know an algorithm that runs in polynomial time. But there's a big jump between that, and then someone asked me later on about the Artificial Intelligence. So it occurred to me during the middle of the night that one way to think about it is this:

I want to give an example where maybe there exists an algorithm to decide the 3-SAT problem in polynomial time. But we won't know what the algorithm is. So imagine that we have a problem of size n, and we build, let's say, $(n^{1,000,000})$ -state, some kind of a neural network that has $n^{1,000,000}$ neurons. So this is a polynomial number of things. Just add a few more [zeroes to the exponent]. So now we trained this neural network on lots and lots of SAT problems. You know, we've got great advances now in machine learning theory. Now I feed it a new SAT problem, and how do I know that it's not gonna solve all of them? In fact, in order to prove that, we couldn't possibly train this neural network. In order to show that P is unequal NP, we would have to show that there's no way to train this neural network to do it.

And that's a situation that we're faced with. Right now, when we do train neural networks, we've got something that solves the problem, but we have no idea what the network is doing inside. And that might be a way to think about the difference between an algorithm existing and an algorithm that we know what it is. Ok. Do you have a comment on that?

Zlatuška: If I may. **Knuth:** Yeah.

Zlatuška: Could you extend this problem to the problem of human mind? The human mind and limits of human mind?

Knuth: Okay.

Zlatuška: Because that *laughter* obviously... That's obviously similar problem.

Knuth: Yeah.

Zlatuška: So what's your idea about limits, the capacity of human mind? You know, the ideas like mysterianism, and the existence of problems which are inaccessible to human mind?

Knuth: In that case, we actually have rigorous proofs, because there are incompressibility theorems. So we know that there are things that cannot be made small, and so the human mind can't possibly understand something that has more states in it then there are, well, let's say, protons in the universe or something like this.

Zlatuška: So, something between man and the God.

Knuth: Yeah. But in one of my papers, I have a number that I called... I don't know, I forget... I shall call it Super K, which is also the name of a breakfast cereal in America, but anyway, I needed a special font in order to do it. Like I wanted to print it only in color, but if you look with a magnifying glass at the paper where I talk about Super K...

Anyway, this was a number that was, I forget, it was something like $10\uparrow\uparrow\uparrow\uparrow$ 3, which is defined in terms of the [Knuth] arrow notation. Anyway, I was once giving a talk at Livermore Lab where they were trying to impress me by how big their equipment is, so I said: "Well, here, let me show you some big numbers that are bigger than you ever thought of before." When you try to think—what is this number Super K—this simply means writing on blackboard that you take ten... let's see... quadruple arrow I don't even remember the thing... Anyway, $10\uparrow\uparrow10$ is equal to $10\uparrow10\uparrow10\uparrow\ldots$ and then if you want to go to triple arrow, then you simply do this that many times. Pretty soon, it gets mind-boggling.

But still, this number is finite, and almost all numbers are bigger than this. laughter You get a little idea that even though computer scientists stick to studying finite numbers, we aren't limiting ourselves too much. There's still a lot of interesting stuff down in... I don't necessarily insist on immortality, I would settle for living this long. laughter

Zlatuška: And if I may misuse the moderator's role, I would like to ask about music a bit. Within Fantasia Apocalyptica, you have lots of quotations from... or sort of inspirations from other authors. And what seems to me really fantastic is the scope. You go from Gregorian chants to authors whose music is sort of dramatic, like Olivier Messiaen. And incidentally to see quotations from Olivier Messiaen and Bruce Springsteen at the same time, well, both of them are favorites of mine, but what seems to me interesting, just to combine those absolutely different styles which are usually considered incompatible. How did you solve this compatibility problem?

Knuth: So come on Friday, but what's your opinion of rap music? *Zlatuška laughs* Because there's a quotation from Eminem as well. *laughter* And of course Dvořák. There are a few notes that are actually original with me as well. I wanted to mention that now before I forget because it ties in with Brno. When I came 24 years ago, my first visit to this part of the world, as my plane was approaching the airport in Prague, I woke up that morning with a melody in my head. And I wrote it down. Probably in the airline magazine or something, but anyway, I wrote it down, and I kept that piece of paper, and I saved it. I sort of thought of it as a Prague theme. Because really, I woke up and here was this melody and I wanted to write it down before I forgot it.

So fast forward twenty more years, and I'm writing Fantasia Apocalyptica, and I came to a point of the piece where I'm trying to represent Chapter 21... I guess I gotta go back. The point of Apocalyptica refers to Apocalypse, the last book of the Bible. I'm trying something new in this piece, which I don't think had been done before. To make a fairly literal translation of an original Greek text and convert it into musical equivalent. So I find more than hundred fifty motifs for things that are repeatedly used in the Greek text. The same sequence of motifs occurs in my piece...

Zlatuška: You consider yourself Wagnerian?

Knuth: Yes, Wagner had motifs, but he never told anybody what they were. So the differences is...

Zlatuška: He wasn't university teacher, so...

Knuth: For example, the motif for war is staccato. The motif for angels is an arpeggio. keyboard playing The motif for God is a three-note melody. keyboard playing If you're referring to the first person of the Trinity, you emphasize the first note. keyboard playing If you're emphasizing the second person of the Trinity. keyboard playing And guess what the third person of the Trinity. keyboard playing And the motif for the devil, in the book of Revelation, there's also a [sic] Anti-Trinity. And so this is keyboard playing the devil. And there's the first person keyboard playing and the second. The Book of Revelation has about ten thousand words in Greek, almost exactly, and I didn't just go word by word because I want it to be good music.

I use this as the constraints to say what kind of good music is suggested by this pattern of motifs. And when I get to Chapter 21, the story talks about the New Jerusalem. Here's a Golden City coming out of the sky, and that's where I got to use my Prague theme. I'm not sure which... I'll just play it instead of trying to find the absolute best combination of voices. So it goes like this: keyboard playing So, da-da-da da-da-da da-da-da da. That's what I wrote down in the airplane. Then it goes on a few more bars, it uses some of the chord progressions of Dvořák's New World Symphony. And New World Symphony is like New Jerusalem. So this is a little bit of music here, that...

Zlatuška: So this is actually what we lose for not having international airport in Brno. Because if Don landed in Brno, that could have been Brno theme.

Knuth: So if I had never been invited here in the first place, who knows what would have happened. One other footnote to yesterday and that is: There was a question where I referred to Stuart Russell. Now I can give you the definite reference because I recommended that you watch this video. I think it's five minutes long, and it's easy to find. So Stuart Russell... writing on blackboard And there's been a lot of follow-up on it, but this came out not quite two years ago, and it's called Slaughterbots. It's sort of a mind-changing video that I recommend everybody to look at. It's very important, I think, that we should ban autonomous weapons, and there are thousands of computer scientists who have signed declarations and are actively trying to make sure that the dangers are minimized.

So that was end of footnotes to yesterday, now I'm ready for new question.

Zlatuška: So I guess... Have you ever burned out?

Knuth: There was a period about 1990, where I was feeling kind of low and I actually I got a little worried about it, because one of my uncle's had gone through a period where he was, I don't know, not getting along with anybody else in the family and so. So when he got old, and I said: "Oh-oh! Maybe I'm inheriting some mental problem." Anyway, so I went to see a psychiatrist, and he said: "Have you ever heard of a Type A personality?" He was a great doctor, he showed me his textbooks. Instead of telling me, sort of preaching to me, he said: "Here, look. Here's a book I was giving you. If you look in this chapter, you'll see something that describes you to a T (???)."

I learned from that how to reduce the stress, and so it was 1990. So how old am I? 50... 52 years old. And I realized why physical education was a required subject in college. I had never done much exercise, so I started swimming in 1990, and very quickly I was happy again.

But there was a time when... Well, I work sort of 24/7, in some sense, but it's fun. Mostly. I have to psych myself up in the morning, and once I get into something, then it's hard to stop again, so that's why I have to turn off like somebody mentioned yesterday. ... Well, how do I describe my daily schedule?

I have a very peculiar scheduling algorithm. You wake up in the morning, and you have to decide what you're gonna do next. The algorithm that I finally decided to work the best is that I always do what I hate the most. Of all the things that I have no good excuse for not doing, which one would I rather not do. And that's what I choose. So all the time I'm working on something I don't wanna do, in a sense.

On the other hand, at the end of the week, I've got stuff out the door. And I'd have to do those unpleasant things anyway, so as long as there's no good reason to procrastinate anymore, that's what I choose to do. Somehow that turns out to be a better scheduling algorithm than the other way, where, "what is the most fun all the time?"

I noticed that my mood, when I start feeling bad, is really if several weeks have gone by, and I've never had a time to do anything creative. There's something, there's some urge that says: "Prove a theorem or something!" So if there's too much busywork, I can stand that for a little while, but after three weeks I have to try to do something new. I can't explain why that is. But that seems to be true.

Zlatuška: I would just add to that Stuart Russell, for anybody interested: There is a new book, which is Human Compatible by him and that was published yesterday.

Knuth: Oh, yesterday! *smiling* ... Human...

Zlatuška: ... compatible. Staying... Artificial Intelligence and the problem of control. And that looks pretty much as this topic.

Knuth: Very good.Zlatuška: Tabs or spaces?

Knuth: Tabs or spaces? smiling You're going back to that question again? Knuth

laughs

Zlatuška: Oh! Okay, it was... ???

Knuth: No, I have a question: Why does [sic] people ask about tabs or spaces? To me, when I make somebody else's code in order to read it, [and] it comes in with tabs, it confuses TFX. The Tab prints as a letter gamma, so I have to go through it, get rid of all the Tabs, and change to space, and then I can print the file.

Zlatuška: That's a proper answer, after which nobody would ever use Tabs.

Knuth: laughing Ok.

Zlatuška: Ok. Biggest challenge of becoming a good programmer?

Knuth: Biggest challenge of becoming a good programmer? Well, being born a geek. I know, some people think that it's just a matter of motivation and trying harder and keep educating, read and write right books and so on. Well, that might be true, but my experience is different.

My experience is that I know many many intelligent people who are, no matter how motivated they are, I don't think they'll ever be a good programmer, and conversely, I know that I'm never gonna be good as a programmer of a quantum computer. There's something about the way my brain works that gives me a great intuition for the classical computer, but not for quantum computing. And it's not a matter of good or bad or trying or harder or not being motivated. It's a quirk that I happen to [have] been born with one of these peculiarity [sic].

Zlatuška: What's your idea in this context about sort of mandatory courses of programming within elementary school? If it is true that not everybody can be a programmer.

Knuth: No, no! You left out the word "good". laughter

Zlatuška: Ah, ok. laughter

Knuth: I think people can become a programmer [sic], but dogs can walk on their hind legs.

Zlatuška: So you feel that there's not enough of bad code.

Knuth: What I'm saying is: If you somehow realize that you've been born a geek, then you owe it to the world to you use that talent because the world needs this talent. That's the way I look at it.

Zlatuška: Vim or Emacs... That was also yesterday. I am not quite sure...

Knuth: Vim or Emacs. Yeah, yeah, ok. I just did admit to using Emacs, but I learned how to use vi enough to know how to quit. laughter That was the hardest first lesson. I mean, I had to go Control-Q or something, I don't know.

Zlatuška: Do you think that we are living in a computer simulation?

Knuth: Do I? *smiling*

Zlatuška: Is your God a programmer?

Knuth: It's hard to tell. laughter So whether I think so or not doesn't matter. The whole question about how far Artificial Intelligence could go: It certainly could go to the point where it has decided that we should have this gathering today.

Zlatuška: There's one consequence of living in computer simulation because that means that the world would be only processes which are computable.

Knuth: And finite, yeah. So, for example, the game of life can simulate any such thing. You have any universal scheme, then it would represent the lecture I'm giving now. As one very special case, it would represent Stuart Russell's book, etc. It would represent all discordant ways to write the Fantasia Apocalyptica.

Zlatuška: What was the worst code you have ever seen?

Knuth: What was the worst code I have ever seen? Hey, I love this! *smiling*

Zlatuška: All you down to ??? level.

Knuth: No, no. I had this student in the 70s who was not born a geek *laughter*, but he came to Stanford from, I think, West Point. Anyway, he was trained as a soldier, and he would follow orders. He was not a Ph.D. student; he was master student. But he did a master's project which was to automate the system that we had in the Computer Science department for sharing technical reports with other universities. So we had to make subscriptions to others, and then if they send us their reports, we send them our reports gratis. We had this large database, sort of... data processing problem. So we needed somebody to do that, and I gave that to him as a master project. He wrote a system that was going to handle the technical parts. I saw the thing, I was busy at the time, and it looked like the right number of pages and so on. And it had a sample where he had taken a couple of... a small database with a few reports, and it gave the right report, so I gave him A on it, and he got his degree. Well, that was in June.

Then in July, the secretary called me: "So Don, we're having a little problem using this system." So I went, and that was one of my first trips to the Stanford AI Lab, where they had special computers there. I started looking at the code that he had written. I got to page three or page four, and it was an example of shellsort, a sorting routine, but it was implemented... It was the first time I had seen a program where I could change one character in the program and make it run hundred times faster. *laughter* And the thing was, the variable should have stepped by H instead of by one. So he wrote shellsort so that it was just a plain old insertion sort. And clearly, he didn't understand that. So I made a copy of that page: "Hey, I gotta show this to my students next ???!" Then I turned to the next page, and he did a binary search on that.

So every page I turned to, I saw a new kind of programming error that I had never seen before. *laughter* And finally I looked at the whole way he had organized the thing. It's really hard to explain, but the text editor that we used in that day—this was way before Emacs—it would start out with an index page, so text editor would always prepare automatically a table of contents at the beginning. He had assumed that text editor would always put all of the whole database into alphabetical order in just the way that you could read it, knowing that there would be line breaks or anything like this. So it was impossible to fix the program. You couldn't possibly write a program that was assuming that you were gonna use the text editor's database for table of contents to do the data processing. So that wins my vote.

Zlatuška: Well, given the fact that programming is actually about giving orders, and maybe he studied military academy. That means some privates should never be given the ability to issue orders.

Do you have any experience with psychedelic drugs, like Richard Feynman? Altered state of consciousness. LSD, psilocybin.

Knuth: No, I'm glad that I didn't, nor did my kids. As far as I know. Near-death experiences? No.

Zlatuška: Any problems you gave up on trying to solve?

Knuth: Any problems you gave up on trying to solve, yes. Many times, In fact, I certainly thought that I had proved that P was unequal NP rather earlier. And after I had solved it, then I wrote it up, and finally, everything disappeared, just about as I was writing the last line of the paper. I realized that it was hopeless.

I can remember the first time I actually solved a problem that I had given up on, which was a kind of a breakthrough. As a student, I had tried various things, and I sort of had a "give it five days and, if you haven't got any ideas, then let it go." But once, I let it go, and the next morning, I woke up, and I said: "Wait, what if I tried this." When you do work on a problem and fail, there's one trick you can try, and that is: Imagine somebody sent you email or letter or knocked on your door, and said "So??? I just solved that problem." And then use: "Oh, I bet I know how he did it." Somehow, if you think the problem can be solved, sometimes it helps you solve it.

But the one that was most dramatic for me, I guess, years ago, when I started, about one third of Computer Science was a study of programming languages, and now my collected papers collected in eight volumes, and one of those volumes is all the papers that I wrote about programming languages. There's a journal called SIGACT News that has book reviews, and they have a page in there saying: "Here are books that we've received. Would you like to review them?" And this collection of my papers on programming languages was on that list for a couple years. Nobody wanted to review it. So nobody cares about this although this the hottest thing in Computer Science when I started.

I worked on a problem that was called parentheses languages. So I think everybody still knows what a context-free grammar is. A way of defining a language in terms of productions. But some context-free grammars have the property like nested parentheses. So if you look at the language, half of the characters are like left delimiters, and half of the characters are like right delimiters. And every string in the language has nested left and right delimiters, properly nested. So the question is: "Somebody gives you a grammar. Is the language that it generates a parenthesis

The grammar won't show any matching between these, but can you somehow look at the grammar and figure out yes or no? Is it really gonna be possible to do this? That was an open problem, and I worked several weeks on it and finally gave up. But then, I think a week later, the key came to me how to solve it. And so, at that point, I was quite delighted to have the solution. I was able to use the insights I got from that when I was doing other work later on, but as far as I know, nobody has ever read that paper.

Zlatuška: Do you solve many programming problems or create music when you

Knuth: Do I... solve many programming problems, create music when I sleep? So,

With respect to the Fantasia, it was kind of interesting that after I started working on it, I had the feeling that the music had already been written, and I just had to listen for it. I don't know, out of body experience... it's like I was channeling in a way, that it was there. It did feel that there was a muse helping somehow.

Now, with respect to programming problems, it goes the other way. If I'm trying to figure out how to solve a programming problem, I can't go to sleep. So I need somehow to either solve it or forget it. But when working on a difficult problem, I usually fill up sheets and sheets of scrap paper, and so I can't keep it all in my head, but I have to write down a whole bunch of stuff in order to get it into my brain. But when it comes to the point, when I can actually think about that problem when I'm swimming, then I know I'm about ready to solve. My brain has absorbed enough so that I've learned how to go from baby steps to giant steps in this territory, the problem domain. So there is this mysterious thing that takes place once I'm ready to do it.

So I always say to my grad students... They're working on the thesis, and I realized early on that was a good idea that they should keep. Every week when we would meet, they would write down in detail what they had been thinking about that week and what was on, what do we know, what don't we know. Then after the problem is solved, you can look at that, and I can use that to prove to them that they solved a hard problem. Because once you solve the hard problem, you think it was obvious, I didn't do anything. But once you see how many hurdles you actually got through... This was good.

Zlatuška: Also yesterday you mentioned that you are Platonist with respect to mathematics.

Knuth: Yes.

Zlatuška: Are you Platonist with respect to music? Music is up there and...

Knuth: The music of the spheres. There's a question. Why is it that some music I can hear ten times and next time I hear, it's just as if I never heard it at all.

Zlatuška: This happens to some students with mathematics. laughter

Knuth: With mathematics as well, yeah.

But, for example, in the old days, when you have CD, I mean long-play records, they would always have to add to the piece you wanted, they had to add something else to fill it out. So I have something by Brahms, and then the publisher added a piece by somebody named Bax. B-A-X. I've heard the piece by Bax as many times as I've heard the piece by Brahms because it's hard for me to get up and turn off the record player. However, I'll never recognize the piece by Bax the next time I hear it. So there's something different about Brahms's music and Bax's music, and I haven't been able to reverse-engineer that at all.

My piece [Fantasia] has parts of it that involve more less random elements. And the question was, well, if you just have random music, does the human brain somehow learn to do it? Well, it didn't work with Bax's music, but there are parts of it where I based on Morse code. I had to make a decision how to spell some Greek words. It turns out that in Ancient Greek, they had a notation for absolute pitch, so you could spell a Greek word just by playing those notes. Let's see if I can find... There's a place in Chapter 2 where the name Jezebel comes out. So Jezebel in Greek is iota, epsilon, zeta, and so on. I think it comes in here... Yeah. *music playing* That's Jezebel. It's kind of ???. Now Jezebel was a prophetess, and the motif for prophet is contrary motion. And so after I play Jezebel, then I play it contrary motion. But anyway, that's random. Still, our brain gets it in, we find ourselves humming. Jezebel, even though there was no reason why we should be able to remember that melody.

So I'm not sure to what extent you can take random elements, and they actually become warm and somehow have a personality. On the other hand, if you have no randomness whatsoever—musicians found long ago—that if you go straight one two three four, one two three four, exactly right, it loses life. You have to go a little bit before the beat, a little bit after the beat in order for music to come to life. And I tried the same experiment with font design. So instead of drawing a letter precisely, I would wiggle some of the points a little bit, and then the alphabet seemed to have a personality.

Zlatuška: Software patents. Software patents, can you elaborate your stance regarding software patents. Patenting software. The second from bottom.

Knuth: Second from bottom? Stances regarding software patterns.

Zlatuška: Patents. Intellectual property.

Knuth: Patents! Ok, aha. Software patterns is another thing, okay software patents, right.

I think people deserve protection for their ideas, but not if just the ideas are trivial. So a great number of software patents were something that we would expect any student to do on an exam, but a lawyer—a patent lawyer not being a geek—wouldn't know how to distinguish those. And so there was a time when people went and tried pro bono make a patent on every trivial idea. So that people wouldn't be able to make us pay them every time they use this trivial idea.

When I wrote TeX, I didn't need to get permission for any of the ideas that I use, the trivial ideas that I used in TeX. But after intellectual property rights got more and more complicated, it might very well have been impossible to write TeX twenty years later.

So when it comes to a substantial piece of software, like the undoing mechanism in Photoshop or something like this, I think this really deserves patent protection for a limited amount of time, but not in perpetuity. That's my general feeling about patents in a nutshell.

Zlatuška: What's your favorite music band?

Knuth: My favorite music... band? laughter I remember The Beatles. But lot of the music after that sounds like noise to me.

Zlatuška: Some of the exercises in The Art of Computer Programming are known to be open research problems. Has anyone ever contacted you that they have solved one of them while reading the book?

Knuth: Yes, I think I mentioned that yesterday, but it's not that uncommon.

Zlatuška: And you don't pay actually...

Knuth: No, no, I only pay...

Zlatuška: It became closed problem, so it was an error...

Knuth: It's stated there that if you find a better answer, you don't get money, you get glory instead, and so instead I mention your name. But if you correct an error, I silently correct it as if I had known it.

Zlatuška: What would you like to redo?

Knuth: What would I like to redo if I could? I would base TeX on decimal instead of binary at the lowest level. TeX is based—at the lowest level—on a scaled point, of which there are 2^{16} scale point to weigh a point. So internally, everything is kept in binary but is communicated to the user in decimal. So the user doesn't get to see... This leads to strange results. You know, you say one third, and then you multiply it by three, and you get 0.999 instead of one. So that would have been better to do that.

Zlatuška: Do you still use T_FX often?

Knuth: *giggles* Well, let's see... I haven't used it since last Friday because I've been in other town.

Here's a... Oh, I see. You're not raising your hand, you're raising the camera. Ok. But other people out here who...

Zlatuška: Anybody from the audience? **Someone:** Can you play something?

Zlatuška: Can you play something?

Someone: Like a longer piece.

Zlatuška: Longer...

Someone: Anything you like.

Knuth: Well, I only have this music here. So, choose a random chapter.

Someone: Uh... Seven? *laughter*

Knuth: Seven. Ok, so seven is the chapter where the saints come marching in. And...

Zlatuška: 3:16? Knuth: What? **Zlatuška:** 3:16?

Knuth: So there are different motifs here. I'm trying to see which are the easiest to explain. Well, ??? mentioned that [it] has lots of different styles, and since this is about the Apocalypse, one of the styles had to be calypso. So in Chapter 7, we get calypso: music playing Now, that's trying to imitate an organ. Let's try to just do a piano. I don't know how to do this here. Voice... Voice "church [organ]"... Let's change other voices... How do we go down?

Szaniszlo: What kind of voice?

Knuth: Just take piano, for example. Grand piano, here we go. *music playing* Now this last thing is *music playing* Harry Belafonte, so "run Venezuela". *music playing and Knuth singing* "She ran with the tailor." *laughter* That's what I'm singing to myself when I was writing this piece. However, this is, of course, taking place as the saints come marching in. *music playing* So at this point there's a grand shout, comes along, and *chord plays* at the is called *music playing*. That's God. *music playing* Next is: *music playing* This is the motif for the lamb, one of the principal characters, and it's supposed to sound like baa baa. *music playing* So the scene we have here is that the saints go onto this hill, and then there are 24 elders, and the elders form a chromatic scale of 24 notes. *music playing* Besides the elders and other characters are Seraphim, and there are four creatures, and the theme for the four creatures is *music playing*. And you can do this: *music playing*.

Is that enough? applause

Zlatuška: By the way, is it difficult to actually play something for organ just on piano? If I understand...

Knuth: Yes, but it's also very difficult... It's also very difficult to play this on the organ. Jan [Rotrekl, the performer of the Fantasia Apocalyptica], the organist, has had to work very hard in order to do it. When I wrote this piece, I didn't hold back. I knew it was the only piece I was ever going to write, and so I didn't bother to simplify it. I wrote what I thought I wanted to hear, and so he has to play what I wrote.

Zlatuška: With (???) Don, when I asked him [Don] whether we can perform this, he mentioned that the organist who did the world premiere was ill and unable to play that, but there is a proof that it is playable. *laughter*

Knuth: *laughs* Yes. I can play it, but not at speed. Well, the organist who did work on it actually fell in love with—I'm glad to say—and he wrote me a couple weeks ago saying that you he's feeling withdrawal symptoms, he wants to play it again.

Zlatuška: Anybody else?

Someone: I have a question. I have read about your WEB and CWEB projects. Do you think that the problem in Computer Science that you were trying to solve with this projects is already solved by maybe new programming languages or different approaches towards documentation? Or do you think that the projects are still relevant today? And maybe a follow-up: If you had ability to write CWEB or WEB again today, would you do anything differently than you did before?

Knuth: To me, it's one of the things I love the most about my life, is that I can write programs in CWEB. However, in order to do that, I'm also living with the fact that the implementation that I have had never been tuned up to a great programming environment. So, for example, I start a CWEB program, and I always type a few things. Like I always type a line that says: |@...| at-sign asterisk index period, and I put that at the bottom. It's a little bit of a nuisance, but in fact everything in life has some small nuisances, and so as long as something is working for 97 % of the time, I'm not gonna spend much time figuring out the 3 %, but a mature program, they start fixing up the 3 %.

So I use CWEB knowing that it was a quick and dirty implementation, but it still does almost everything that I want, exactly as I want. And I have an Emacs interface to CWEB. My wife will tell you I come out of my office several times a day saying: "It's so fun programming in CWEB!"

And the reason is that as I'm doing it, it seems to me that I'm combining the formal and the informal aspects of the program the way they ought to be. In order to understand any complicated technical subject, one of the main tricks of it, of a person who writes about Computer Science, mathematics, is to say everything twice: once formally and once informally. Maybe three times, but from different perspectives, but you don't only give a formula, but you say what the variables in the formula mean. You write a program, you not only declare something to be ??? and a variable. You

say what has an invariant relation: This is the number of nonzero elements in the array or something, but you combine formal and informal.

And that's the way CWEB works. It breaks down a program into a small number of pieces that fit together in a small number of ways. But each module of the program is a combination of informal-which you write in natural language-and formal-which you write in whatever formal language you're using. In CWEB, it's C, but there are many different flavors of WEB.

So I really think there's nothing else anywhere near as good as my approach, but I know that there are many ways to refine it, and I'm not interested in actually pursuing the refinements because it's good enough for me. On the other hand, most of the world writes... you look on the solid programs on GitHub. You'll find that they're probably not using CWEB, but there's a certain style that's become... I don't know, I don't like C++ program because... it's so ambiguous. Each C++ compiler does different things with these programs, and so when people... If somebody sends me a program in C++, I don't know what subset of the language they're using. There's all kind of things going on automatically, and the programmer knows what she's doing, but their reader doesn't know which subset is there, so I don't care for that.

But let's suppose we look at a typical module that we get, that's written in C, and it's a style of programming that people have learned to read and maintain, and so it works. I can say: "Oh yeah, let me rewrite your program in CWEB, and you'll see that it actually not only is better, it's more reliable, and you'll realize that certain features were missing because of the discipline of literate programming." However, I don't believe I'll ever convert the world with this. It'd be like saying Esperanto is a much better language than English, so therefore let's abolish English. English works well enough so that some ideal language isn't going to displace it.

Zlatuška: There's question by Tim: Paul Erdős spoke of book in which God kept the most elegant mathematical proofs in the same sense of divinity that are key??? I would extend this also to the most elegant programs, but...

Knuth: I read a very strange paper recently where they asked people to compare algorithms to music. One group of subjects, they were supposed to compare algorithms to music, and another group of subjects was supposed to compare the algorithms to art. Where was this paper? It was one of the papers I read in the last three weeks. Anyway, these people presented keep sort, binary search, ... they took five algorithms that they thought were classic, and then they showed the subjects several pieces of music, and the subjects were supposed to say: "Okay, now match this algorithm to this piece of music." They found—maybe a hundred subjects—that there was a fairly good correlation, it wasn't random permutation of this matching between algorithms and music. But then they did another group, and they showed them five paintings, and they were supposed to again associate this with the painting, and that didn't work at all. But maybe they didn't choose the right paintings. I don't think all kinds of art are equivalent. Certainly, there's a quantitativeness to music that's similar to Computer Science.

Zlatuška: Suppose we create a technology to faithfully copy and simulate working human brain. Would you want to continue living as such a simulation? That's apparently the Kurzweil's idea.

Knuth: So there's a line in... That's a Gershwin's song Ain't Necessarily So. And it says something like Methuselah lived nine hundred years. Methuselah lived nine hundred years, but who calls that livin' if no gal won't give in to a man who was nine hundred years. So there's more to living than thinking.

Knuth: Your favorite fractal?

Knuth: My favorite fractal. Well, it has to be the dragon curve because that was the first one I knew about. The dragon curve—you can look it up—but basically, you take a long sheet of paper. Like thin sheet of paper, like we used to have adding machine tape or something. And you fold it in half, you crease it, you fold it again, and you fold it again, so each time it gets half as big as before. And you've got creases in the paper. Then you open it up, some of the creases go down, and some of them go up. It makes a pattern. An interesting pattern that also turns out to be equivalent to the Legendre symbol of minus one over anything (???). Anyway it's a pattern. You open up the paper, and you make all the bends go ninety degrees. So it'll go like this *drawing on blackboard*, and then go like this... something, left-right, left-right. You can round off the corners if you want, but this is the idea of dragon curve.

It turns out it never intersects itself, and if you take four of them, and start them out... I probably didn't drop correct dragon curve, but if I take four of them, it will cover the entire plane, with four of them. I had a lot of fun proving that theorem in the 80s. And I was really proud when that theorem was picked up in Russia in Quant magazine, which late sixties, of course, there was the iron curtain then, and they illustrated my theorem with four colors in this magazine written for high school students in Russia. It says, you know, I knew enough Russian to translate it, it said: This is a difficult theorem, it was proved by Donald Knuth.

Now we see Czech version of... my books.

Zlatuška: I think the topmost question was already tackled.

Knuth: Are there any other questions from the...

Zlatuška: From the floor?

Someone: What kind of organ do you have at home? Is it a pipe organ? Mechanically controlled or a digital or electronic...

Knuth: Yes, it has 850 pipes or so, and the website explains it all. If you go to my home page and look under pipe organ. It was built by a firm called Abbott and Sieker—they're both dead now—but they used to make about four organs a year. And it has 17 ranks of pipes, and I have a major exercise in Fascicle 5—which is coming out next month to say—how many different sounds can you make on this organ that have exactly five pipes going, or exactly six pipes going, or so on. I found it to be quite a fascinating exercise. So if you want to know more about the organ, online has the specs, but also then you've got to buy the book, [to] look at this exercise about that organ that I have.

Someone: So I assume that you also prefer pipe organ, mechanically controlled, instead of say, digital organs.

Knuth: I am sorry, I don't understand the question.

Zlatuška: You prefer classical organ to digital?

Knuth: Oh, I see. Yes, absolutely. I heard a pretty good digital organ in England in July, but it's quite rare. They make good digital recordings of organs, most of the organs that I've ever had a chance to play. Even though it's in a great building, and you had the reverberation and everything, it just doesn't match.

I come from a part of the United States where it was illegal to some—it's called America's Dairy State, America's Dairyland—and so it was illegal to sell margarine instead of butter in our state. If you wanted to get margarine, you had to go to Illinois *laughter* to buy it, it was a little cheaper. But I look at butter versus margarine, that sort of has the difference between pipe organ and electronic organ. But also in the early days, before we had good resolution in fonts, there was good printing—butter—and there was the kind that we could get in our lab in the early days—margarine.

Zlatuška: You originally wanted not to have your organ built by some American company, but you imported from some Nordic country. Do you regret that did not work? Or...

Knuth: I heard some really beautiful organs in Denmark, and I inquired about having a Danish organ, and this was in the 70s, there was only one Danish organ in America at that time, it was one in Boston. So I had talked to the builder, and then I found out that there was no way... I had a limited budget, and according to Danish law, the only way I could buy this organ was to agree that the price was indeterminate until after it was built because by Danish law whatever the Union workers wanted to get for it, had to be the amount that was done. So they couldn't get me a fixed price for it, and I might have gone broke, so I was unable to do that.

Zlatuška: So your love for organ was not that big that you would get broke because of it? laughs

Knuth: The organ that I finally bought cost \$35,000. People were paying that for a house in those days. Now you get a house for \$35,000, it's impossible.

Zlatuška: What will happen with your organ when the lease of your house expires?

Knuth: Who knows. But it has only existed in this room. People could unscrew it and reassemble it somewhere.

Knuth: Somebody else? Ah, over there.

Someone: You mentioned once that in Super Bowl game the crowds, they have flags with... showing 3:16 on it. I've always been puzzled what does that have to do with football game.

Knuth: There are people who flaunt their religion. And the most famous verse in the Bible by its number is John 3:16, which is said to be the gospel in a nutshell. It says the God loved the world in such a way that He gave His only child, and so on. So that's a very famous verse of the Bible. People think that by putting that number up that will give publicity, to make people look up this verse and they will suddenly realize that this should be their religion. At football games, it just happened that the cameras, the camera crews survey the crowd, and so this was a way to get advertising for whatever slogans you wanted to do. And a certain group of people started doing

Then there were jokes based on. I mean, there was in baseball game... what was his name... a guy from the Boston Red Sox, but anyway his batting average was 0.316, and his nick, his first name was John. So people hold up a chart saying "John! 0.316!" And this was to be a satire on this phenomenon. But it was something that sort of grew like... we have bumper stickers, slogans that people put on their cars and things like that.

But the fact is that this number, the verse got popular by its number, and I used that later when I wrote this book called 3:16 because I wanted to use a cross section of the Bible in order to understand the complexity of it and have some way of sampling. So I used this in order to get a good cross section of not the Bible itself so much, but all the secondary literature about the Bible. So there have been 100,000s of books written about the Bible, but I could go into a theological library, and most of those books have an index in the back, and they'll say which verses do I refer to. So I go through, and I find out, oh yeah, I only have to read a dozen pages of this book, and I can see what it says about Genesis 3:16 and what it says about Revelation 3:16.

By the way, Revelation 3:16 is one of the verses that's here [in Fantasia Apocalyptica], so I might as well go to Chapter 3. flipping pages The verse Revelation 3:16 says something like "because you are lukewarm, neither cold nor hot, I will spit you out of my mouth." The message is that God prefers atheist to people who don't care at all. So when I do verse 3:16, I had a little fun in the music. I used time signature 3/16, which is three sixteenth notes to a measure. And then it says: "I will spit you out of my mouth," so on the organ, we have here in the church on Friday, has a pipe called the spitzflute, so we use a spitzflute for this spitting out of the mouth. So it goes anyway music playing and then spitz! music chord Listen for that on Friday.

Zlatuška: I guess that we basically finished the time allocated for this. There will be the question: "How are you today?" but that will be postponed for next time. Maybe if Don wakes up in the night, maybe he starts with another sermon in the Church, but... Now, that was just a joke.

So thank you very much, I believe that... *applause* Thank you very much for coming, for having these sessions with us. An invitation to everybody for Friday, 7 p.m. in the Church of Jesuits, where the piece Fantasia Apocalypsa [sic] will be performed.

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Tomáš Szaniszlo xszanisz@fi.muni.cz