Heap Sort in \TeX

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September 1992

Abstract

Sorting in plain \TeX is implemented via heap sort. The heap sort algorithm is explained and the encoding given.

Keywords: (heap) sort, array addressing, plain \TeX, macro writing, education.

1 Introduction

A \TeX encoding of the heap sort algorithm is given. A simpler user interface and examples of use, also with respect to lexicographic sorting, will be treated in Sorting in BLUe, to come.

2 Example of use

\[
\begin{align*}
&\texttt{\let\ea=\expandafter %28/9/92} \\
&\texttt{\newcount\n} \\
&\texttt{\newcount\l} \\
&\texttt{\newcount\r} \\
&\texttt{\newcount\i} \\
&\texttt{\newcount\uone} \\
&\texttt{\newcount\j} \\
&\texttt{\newcount\jj} \\
&\texttt{\newif\ifcmp} \\
&\texttt{\newif\ifgoon} \\
&\texttt{\newif\iflt} \\
&\texttt{\def\heapsort{%data in \1 to \n} \\
&\texttt{\r=\n \heap \i=1 \%} \\
&\texttt{\loop\ifnum\r>1 \exchange\i\r\%} \\
&\texttt{\advance\r by-1 \sift\i\r\%} \\
&\texttt{\repeat} \\
&\texttt{\def\heap{%Transform \1..\n into heap} \\
&\texttt{\l=\n\divide\l by2 \advance\l by1 \%} \\
&\texttt{\loop\ifnum\l>1 \%} \\
&\texttt{\advance\l by-1 \%} \\
&\texttt{\loop\ifnum\l>1 \sift\l\n\%} \\
&\texttt{\repeat} \\
&\texttt{\def\cmpval#1#2{%#1, #2 counter variables} \\
&\texttt{\xdef\aone{\csname\the#1\endcsname}} \\
&\texttt{\xdef\atwo{\csname\the#2\endcsname}} \\
&\texttt{\cmptrue\ifcmp\aone>\atwo
\cmpfalse\fi} \\
&\texttt{\def\exchange#1#2{%#1, #2 counter variables} \\
&\texttt{\edef\aux{\csname\the#1\endcsname}} \\
&\texttt{\xdef\csname\the#1\endcsname{\csname\the#2\endcsname}} \\
&\texttt{\xdef\csname\the#2\endcsname{\csname\the#1\endcsname}}} \\
\end{align*}
\]

yields 314, 27, 1.

3 Heap sort

The process consist of two main steps

- creation of a heap
- sorting the heap

with a sift operation to be used in both.

What is a heap? A sequence $a_1, a_2, \ldots, a_n$, is a heap if $a_k \leq a_{2k} \land a_k \leq a_{2k+1}, k = 1, 2, \ldots, n \div 2$, and because $a_{n+1}$ is undefined, it is defined that $a_2 < a_{n+1}$ is true. A tree and one of its heap representations of 2, 6, 7, 1, 3, 4, is displayed below

\[
\begin{align*}
&\begin{tikzpicture}
&\node{6} %\node{7} &\node{2} &\node{3} &\node{4} &\node{1} &\node{7} \\
&\node{1} &\node{3} &\node{2} &\node{4} \\
&\end{tikzpicture}
\end{align*}
\]

3.1 The algorithm

In PASCAL-like notation the algorithm reads

\[
\begin{align*}
&\texttt{\%heap creation in ’1:n’} \\
&\texttt{\l:= \n \div \n div2 + 1;} \\
&\texttt{\while {\l} \neq \not1 \do \sift(\l, \l, \n) \od} \\
&\texttt{\%sorting in ’1:n’} \\
&\texttt{\r:= \n;} \\
&\texttt{\while {\r} \neq \not1 \do} \\
&\texttt{\exchange(a[\l], a[\r])} \\
&\texttt{\sift(\l, \l, \r) \od} \\
&\texttt{\%sift \#1 through \#2} \\
&\texttt{\j:= \#1} \\
&\texttt{\while {2j \leq \#2} \land (a[j] > a[2j] \lor a[j] > a[2j+1]) \do} \\
&\texttt{\exchange(a[j], a[mi]) \j:= mi \od}
\end{align*}
\]

3.2 \TeX encoding

\[
\begin{align*}
&\texttt{\let\ea=\expandafter %28/9/92} \\
&\texttt{\newcount\n} \\
&\texttt{\newcount\l} \\
&\texttt{\newcount\r} \\
&\texttt{\newcount\i} \\
&\texttt{\newcount\uone} \\
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&\texttt{\xdef\csname\the#1\endcsname{\csname\the#2\endcsname}} \\
&\texttt{\xdef\csname\the#2\endcsname{\csname\the#1\endcsname}}} \\
\end{align*}
\]
Heap Sort in \TeX

The values of $1, \ldots, n$, are sorted in descending order: $1 \geq 2 \geq \cdots \geq n$.

heap

The values $1, \ldots, n$, are rearranged into a heap.

sift

The first element denoted by the first (counter) argument has disturbed the heap, because of the exchange of the first and the last (one higher than the second argument) element. sift sifts this first element through the heap until the heap property holds again.